e-ISSN:2582-7219

INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY RESEARCH IN SCIENCE, ENGINEERING AND TECHNOLOGY

Volume 6, Issue 9, September 2023

INTERNATIONAL
STANDARD
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INDIA

Impact Factor: 7.54

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| ISSN: 2582-7219 | www.ijmrset.com | Impact Factor: 7.54| Monthly Peer Reviewed & Referred Journal |

| Volume 6, Issue 9, September 2023 |

| DOI:10.15680/IJMRSET.2023.0609007 |

ON THE HEPTIC DIOPHANTINE EQUATION WITH THREE UNKNOWNS $5(x^2+y^2) - 9xy=35z^7$

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ABSTRACT : Different sets of integer solutions to the non-homogeneous heptic Diophantine equation with three unknowns given by $5(x^2+y^2)-9xy=35z^7$ are presented in this paper.

KEYWORDS : Heptic equation , Heptic with three unknowns

Non - homogeneous Heptic ,Integer solutions

I. INTRODUCTION

It is well known that Diophantine equations are rich in variety. In [1-6] , the Quintic and Sextic Diophantine equations with three unknowns are considered for getting their corresponding integer solutions. While collecting problems on higher degree Diophantine equations ,the problem of getting integer solutions to the non-homogeneous heptic Diophantine equation with three unknowns given by $5(x^2+y^2)-9xy=35z^7$ [7] has been noticed. The authors of [7] have presented a few sets of integer solutions to the heptic equation considered in [7]. The main thrust of this paper is to exhibit other sets of integer solutions to the considered equation in [7].

Method of analysis

The non-homogeneous heptic Diophantine equation with three unknowns to be solved is given by

$$5(x^2+y^2)-9xy=35z^7 \tag{1}$$

Different ways of solving (1) are illustrated below:

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Way 1:

Taking

$$x = k\,y \tag{2}$$

in (1) , it is written as

$$(5k^2 - 9k + 5)\,y^2 = 35z^7$$

which is satisfied by

$$y = 35^4\,(5k^2 - 9k + 5)^3\,s^7\,, z = 35\,(5k^2 - 9k + 5)\,s^2 \tag{3}$$

In view of (2) , we get

$$x = 35^4\,k\,\,(5k^2 - 9k + 5)^3\,s^7 \tag{4}$$

Thus,(3) & (4) represent the integer solutions to (1).

Way 2:

Introduction of the linear transformations

$$x = (7k - 2)\,v\,, y = (7k - 4)\,v \tag{5}$$

in (1) leads to

$$(49k^2 - 42k + 28)\,v^2 = 35\,z^7 \tag{6}$$

which is satisfied by

$$v = 35^4\,(49k^2 - 42k + 28)^3\,s^7\,, z = 35\,(49k^2 - 42k + 28)\,s^2 \tag{7}$$

In view of (5) , we have

$$\begin{aligned} x &= 35^4\,(7k - 2)\,(49k^2 - 42k + 28)^3\,s^7\,, \\ y &= 35^4\,(7k - 4)\,(49k^2 - 42k + 28)^3\,s^7 \end{aligned} \tag{8}$$

Thus, the values of x, y, z given by (7) & (8) represent the integer solutions to (1).

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Way 3:

Choosing

$$x = u + z^3 , y = u - z^3 \tag{9}$$

in (1) , it gives

$$u^2 = z^6 (35z - 19) \tag{10}$$

It is possible to choose z so that the R.H.S. of (10) is a perfect square and

The corresponding value of u is obtained .In view of (9) , the respective values

of x , y satisfying (1) are found. A few examples are presented in the Table 1

below:

Table 1-Examples

z	x	y
1	5	3
4	$12*4^3$	$10*4^3$
17	$25*17^3$	$23*17^3$
28	$32*28^3$	$30*28^3$

II. CONCLUSION

In this paper, integer solutions to $5(x^2 + y^2) - 9xy = 35z^7$ different from the solutions in [7] are presented. One may search for sets of integer solutions to the other forms of heptic equations with three or more variables.

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