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Mean Cordial Labeling Of Some Graphs

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ABSTRACT: Let f be a function from $V(G)$ to $\{0,1,2\}$. For every edge uvnominate the label $\{0,1,2\}$. f is called a mean cordial labeling if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1, \forall i,j \in \{0,1,2\}$ where $v_f(x)$and $e_f(x)$represent the number of vertices and edges respectively labeled with $x(x = 0,1,2)$. A graph with mean cordial labeling is called a mean cordial graph. In this chapter we study the mean cordial labeling pattern of ladder graph$L_n$and shadow graph of $K_{1,n}$.

KEYWORDS: mean cordial labeling, ladder graph, shadow graph.

I. INTRODUCTION

The graphs under consideration are simple, finite, and undirected. $V(G)$ and $E(G)$represent the set of vertices and set of edges of a graph G, respectively. The number of elements of $V(G)$ and $E(G)$is called the order and size of Grespectively. Tracking system, route design, broadband networks, astrographs, and coding-encoding are just a few of the fields where labeled graphs are used(Gallian, 2022). Cahit introduced the cordial labeling concept in 1987(Cahit, 1987).

Definition 1: Let $f : V(G) \rightarrow \{0,1\}$ be a mapping. For every edge uvnominate the label $|f(u) - f(v)|$. f is called a cordial labeling if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1, \forall i,j \in \{0,1\}$ where $v_f(x)$and $e_f(x)$ denote the number of vertices and edges respectively labelled with $x(x = 0,1)$. A graph that admits a cordial labeling is called a cordial graph(Cahit, 1987).

Definition 2: Let $f : V(G) \rightarrow \{0,1,2\}$ be a function. For each edge uv assign the label $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. f is called a mean cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1, \forall i,j \in \{0,1,2\}$ where $v_f(x)$and $e_f(x)$denote the number of vertices and edges respectively labeled with $x(x = 0,1,2)$. A graph that admits a mean cordial labeling is called a mean cordial graph(Ponraj et al., 2012).

If we reduce the range set of f to $\{0,1\}$ then this definition becomes the definition of product cordial labeling.M. Sundaram, R. Ponraj, and Somasundaram established the concept of cordial product labelling(Sundaram et al., 2004).

I examine the mean cordial labeling pattern of lattice graphs. The sign$\lfloor x \rfloor$represents greatest integer less than or equal to x and $\lceil x \rceil$represents smallest integer greater than or equal to x. Terminologies not described here are used in Harary's context(*Book 1 (Harary).Pdf*, n.d.).

Definition 3: A cartesian product of two graphs $G$and H is the graph $G \times H$ such that its vertex set is a Cartesian product of $V(G)$ and $V(H)$ i.e. $V(G \times H) = V(G) \times V(H) = \{(x,y) : x \in V(G), y \in V(H)\}$and its edge set is

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defined as $E\,(G \times H) = \{\ ((x_1, x_2),(y_1, y_2)) : x_1 = y_1$ and $(x_2, y_2) \in E(H)$ or $x_2 = y_2$ and $(x_1, y_1) \in E(G)\ \}$("Cartesian Product of Graphs," 2022).

Definition 4: The ladder graph L_n is defined as the cartesian product of P_n by K_2 where P_n is a path with n vertices and K_2 is a complete graph with two vertices. Here $V_{Ln} = \{u_i,\ v_i\!:\ 1 \le i \le n\}$ and $E_{Ln} = \{e_i = u_i u_{i+1} : 1 \le i \le n-1\} \cup \{f_i = u_i v_i : 1 \le i \le n\} \cup \{g_i = v_i v_{i+1} : 1 \le i \le n-1\}$.(Sumathi, 2018)

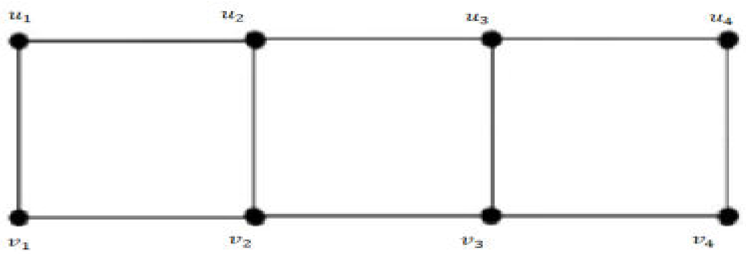

Figure 1: L_4

Definition 5: The shadow graph $D_2(G)$ of a connected graph G is constructed by taking two copies of G say G' and G''. Join each vertex u' in G' to the neighbours of the corresponding vertex v' in G''.(Jayasekaran, 2018).

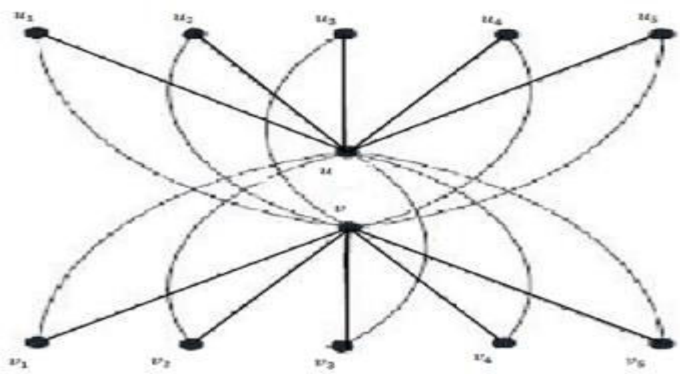

Figure 2: Shadow graph of $K_{1,5}$

Definition 6: Let f be a function from $V\,(G)$ to $\{0,1,2\}$. For each edge uv of G, assign the label $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ or $\left\lfloor \frac{f(u)+f(v)}{2} \right\rfloor$. f is called a mean cordial labeling of G if $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1, \forall i, j \in \{0,1,2\}$ where $v_f(x)$ and $e_f(x)$ represent the number of vertices and edges respectively labeled as $x (x = 0,1,2)$. An illustration using mean cordial labeling is referred to as a mean cordial graph.

MAIN RESULTS:

Theorem 1: Let L_n be a ladder graph, then L_n admits mean cordial labeling.

Proof. Define $f : V\,(G) \rightarrow \{0,1,2\}$ and $g : E(G) \rightarrow \{0,1,2\}$.

Case(i): $n \equiv 0(mod 3)$: That is, $n = 3k$, where k is positive integer.

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We define vertex labeling as follows: $f\ (u_i)\ =\ f\ (v_i)\ = \begin{cases} 2 & 1 \le i \le k \\ 1 & k+1 \le i \le 2k \\ 0 & 2k+1 \le i \le 3k. \end{cases}$

We define edge labeling as follows:

$g\ (u_i\, u_{i+1})\ =\ \left\lfloor \frac{f(u_i)+f(u_{i+1})}{2} \right\rfloor \qquad 1 \le i \le 3k - 1,$

$g\ (v_i\, v_{i+1})\ =\ \left\lceil \frac{f(v_i)+f(v_{i+1})}{2} \right\rceil \qquad 1 \le i \le 3k - 1$ and

$g\ (u_i\, v_i)\ = \left\lceil \frac{f(u_i)+f(v_i)}{2} \right\rceil \qquad 1 \le i \le 3k.$

In this case, we have $v_f(0) = v_f(1) = v_f(2) = 2k$, and

$e_g(0) = e_g(2) = 3k - 1, e_g(1) = 3k.$

Case(ii):$n \equiv 1(mod3)$: That is, $n = 3k + 1$, where k is positive integer.

We define vertex labeling as follows: $f\ (u_i)\ =\ f\ (v_i)\ = \begin{cases} 2 & 1 \le i \le k \\ 1 & k+1 \le i \le 2k \\ 0 & 2k+1 \le i \le 3k. \end{cases}$

$f(u_{3k+1}) = 1\ ;\ f(v_{3k+1}) = 2$

We define edge labeling as follows:

$g\ (u_i\, u_{i+1})\ =\ \left\lfloor \frac{f(u_i)+f(u_{i+1})}{2} \right\rfloor 1 \le i \le 3k,$

$g\ (v_i\, v_{i+1})\ =\ \left\lceil \frac{f(v_i)+f(v_{i+1})}{2} \right\rceil 1 \le i \le 3k$ and

$g\ (u_i\, v_i)\ = \left\lceil \frac{f(u_i)+f(v_i)}{2} \right\rceil\ 1 \le i \le 3k + 1.$

In this case, we have $v_f(0) = 2k$, $v_f(1) = v_f(2) = 2k + 1$, and

$e_g(0) = e_g(2) = 3k, e_g(1) = 3k + 1.$

Case(iii):$n \equiv 2(mod3)$: That is, $n = 3k + 2$, where k is positive integer.

We define vertex labeling as follows: $f\ (u_i)\ =\ f\ (v_i)\ = \begin{cases} 2 & 1 \le i \le k. \\ 1 & k+1 \le i \le 2k \\ 0 & 2k+1 \le i \le 3k. \end{cases}$

$f(u_{3k+1}) = 1\ ;\ f(v_{3k+1}) = 2$

$f(u_{3k+2}) = 0\ ;\ f(v_{3k+2}) = 2$

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We define edge labeling as follows:

$g\,(u_i\,u_{i+1}) \;=\; \left\lfloor\frac{f(u_i)+f(u_{i+1})}{2}\right\rfloor 1 \;\le\; i \;\le\; 3k+1\,,$

$g\,(v_i\,v_{i+1}) \;=\left\lceil\frac{f(v_i)+f(v_{i+1})}{2}\right\rceil 1 \;\le\; i \;\le\; 3k+1$ and

$g\,(u_i\,v_i) \;= \left\lceil\frac{f(u_i)+f(v_i)}{2}\right\rceil 1 \;\le\; i \;\le\; 3k+2.$

In this case, we have $v_f(0) \;=\; v_f(1) = 2k+1$, $v_f(2) =\; 2k+2$, and

$e_g(0) = e_g(2) =\; 3k+1$,$e_g(1) =\; 3k+2.$

In all the above cases the conditions, $|v_f(i)-v_f(j)| \le 1$ and $|e_g(i)-e_g(j)| \le 1, \forall i,j \;\in\; \{0,1,2\}$, are satisfied by the labeling pattern.

Example 1. Figure 3 represents the mean cordial labeling of ladder graph L_5.

This is the caseinvolving$n \;\equiv\; 2(mod 3)$.

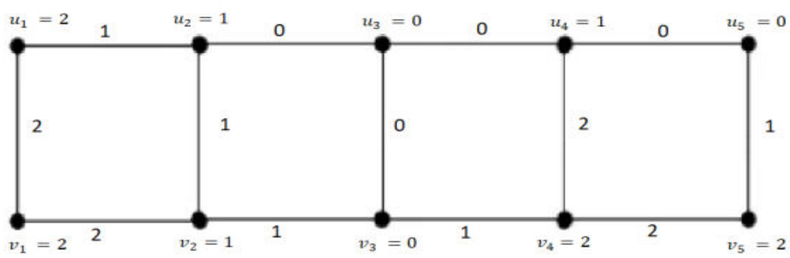

Figure3: Ladder graph L_5

Theorem 2: Let $D_2(K_{1,n})$ be a shadow graph of $K_{1,n}$, then $D_2(K_{1,n})$ admits mean cordial labeling.

Proof: Define $f\,:\,V\,(G)\;\rightarrow\;\{0,1,2\}\;and\;g\,:\,E(G)\;\rightarrow\;\{0,1,2\}$.

Case(i):$n \;\equiv\; 0(mod 3)$: That is, $n \;=\; 3k$, where k is positive integer.

We define vertex labelling as follows.

$$f\,(u) \;= 0, f\,(v) \;=\; 2$$

$$f\,(u_i) \;=\; f\,(v_i) \;= \begin{cases} 2 & 1 \;\le\; \mathrm{i} \;\le\; \mathrm{k} \\ 1 & \mathrm{k}+1 \;\le\; \mathrm{i} \;\le\; 2\mathrm{k} \\ 0 & 2\mathrm{k}+1 \;\le\; \mathrm{i} \;\le\; 3\mathrm{k}. \end{cases}$$

We define edge labeling as follows:

$g\,(uu_i\,) \;=\; \left\lfloor\frac{f(u)+f(u_i)}{2}\right\rfloor \qquad 1 \;\le\; i \;\le\; 3k\,,$

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$$g\ (vv_i)\ = \left\lceil\frac{f(v)+f(v_i)}{2}\right\rceil \qquad 1\ \leq\ i\ \leq\ 3k$$

$g\ (uv_i\ )\ =\ \left\lfloor\frac{f(u)+f(v_i)}{2}\right\rfloor \qquad 1\ \leq\ i\ \leq\ 3k$ and

$g\ (vu_i)\ = \left\lceil\frac{f(v)+f(u_i)}{2}\right\rceil \qquad 1\ \leq\ i\ \leq\ 3k$

In this case, we have, $v_f(0)\ =\ v_f(2) = 2k+1\ , v_f(1)\ =\ 2k$, and

$$e_g(0) = e_g(1)\ =\ e_g(2)\ =\ 4k$$

Case(ii): $n\ \equiv\ 1(mod3)$: That is, $n\ =\ 3k+1$, where k is positive integer.

We define vertex labeling as follows:

$$f\ (u)\ = 0, f\ (v)\ =\ 2$$

$$f\ (u_i)\ =\ f\ (v_i)\ = \begin{cases} 2 & 1\ \leq\ i\ \leq\ k \\ 1 & k\ +1\ \leq\ i\ \leq\ 2k \\ 0 & 2k\ +1\ \leq\ i\ \leq\ 3k. \end{cases}$$

$f(u_{3k+1}) = 2\ ;\ \ f(v_{3k+1}) = 1$

We define edge labeling as follows:

$g\ (uu_i\ ) = \left\lfloor\frac{f(u)+f(u_i)}{2}\right\rfloor \qquad 1\ \leq\ i\ \leq\ 3k+1$,

$$g\ (vv_i) = \left\lceil\frac{f(v)+f(v_i)}{2}\right\rceil \qquad 1\ \leq\ i\ \leq\ 3k+1$$

$g\ (uv_i\ ) = \left\lfloor\frac{f(u)+f(v_i)}{2}\right\rfloor \qquad 1\ \leq\ i\ \leq\ 3k+1$ and

$g\ (vu_i) = \left\lceil\frac{f(v)+f(u_i)}{2}\right\rceil \qquad 1\ \leq\ i\ \leq\ 3k+1$

In this case, we have, $v_f(0)\ =\ v_f(1) = 2k+1\ , v_f(2)\ =\ 2k+2$, and

$$e_g(0) = e_g(1) =\ 4k+1, e_g(2) =\ 4k+2$$

Case(iii): $n\ \equiv\ 2(mod3)$: That is, $n\ =\ 3k+2$, where k is positive integer.

We define vertex labeling as follows:

$$f\ (u)\ = 0, f\ (v)\ =\ 2$$

$$f\ (u_i)\ =\ f\ (v_i)\ = \begin{cases} 2 & 1\ \leq\ i\ \leq\ k. \\ 1 & k\ +1\ \leq\ i\ \leq\ 2k \\ 0 & 2k\ +1\ \leq\ i\ \leq\ 3k. \end{cases}$$

$f(u_{3k+1}) = 1\ ;\ \ f(v_{3k+1}) = 2$

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$f(u_{3k+2}) = 1$; $f(v_{3k+2}) = 0$

We define edge labeling as follows:

$g\ (uu_i\ ) = \left\lceil \frac{f(u)+f(u_i)}{2} \right\rceil \qquad 1 \le i \le 3k+2$,

$$g\ (vv_i) = \left\lceil \frac{f(v)+f(v_i)}{2} \right\rceil \qquad 1 \le i \le 3k+2$$

$g\ (uv_i\ ) = \left\lceil \frac{f(u)+f(v_i)}{2} \right\rceil \qquad 1 \le i \le 3k+2$ and

$g\ (vu_i) = \left\lceil \frac{f(v)+f(u_i)}{2} \right\rceil \qquad 1 \le i \le 3k+2$

In this case, we have, $v_f(0) = v_f(1) = v_f(2) = 2k+2$, and

$$e_g(0) = e_g(2) = 4k+3, e_g(1) = 4k+2$$

In all above cases the conditions,

In all the above cases the conditions, $\left|v_f(i) - v_f(j)\right| \le 1$ and $\left|e_g(i) - e_g(j)\right| \le 1, \forall i,j \in \{0,1,2\}$, are satisfied by the labeling pattern.

Example 1. Figure 4 represents the mean cordial labeling of the graph $D_2(K_{1,5})$.

This is the caseinvolving $n \equiv 2(mod3)$.

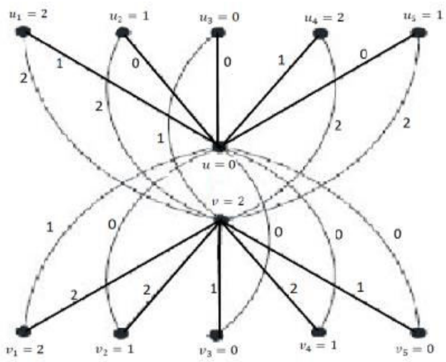

Figure 4: Shadow graph $D_2(K_{1,5})$

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