e-ISSN:2582-7219

INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY RESEARCH IN SCIENCE, ENGINEERING AND TECHNOLOGY

Volume 6, Issue 12, December 2023

INTERNATIONAL STANDARD SERIAL NUMBER INDIA

Impact Factor: 7.54

6381 907 438 6381 907 438 ijmrset@gmail.com www.ijmrset.com

International Journal Of Multidisciplinary Research In Science, Engineering and Technology (IJMRSET)

| ISSN: 2582-7219 | www.ijmrset.com | Impact Factor: 7.54| Monthly Peer Reviewed & Referred Journal |

| Volume 6, Issue 12, December 2023 |

| DOI:10.15680/IJMRSET.2023.0612029 |

Study of Total and Paired Domination of Cartesian product Graphs

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ABSTRACT*:* A dominating set D for a graph G is a subset of V(G) such that any vertex not in D has at least one neighbor in D. The domination number γ(G) is the size of a minimum dominating set in G. Vizing‟s conjecture from 1968 states that for the Cartesian product of graphs G and H ,γ(G)γ(H) ≤ γ(G□H), and Clark and Suen (2000) proved that γ(G)γ(H)≤2γ(G□H). In this paper, we modify the approach of Clark and Suen to prove a variety of similar bounds related to total and paired domination,and also extend these bounds to then n-Cartesian product of graphs A^1throughA^n.

KEYWORDS*:* **-** Domination, Total domination, Paired domination, Vising's conjecture

I. INTRODUCTION

The invariants based on the distance or degree of vertices in molecules is called topological indices. In theoretical chemistry, physics and graph theory, topological indices are the molecular descriptors that describe the structures of chemical compounds, and they help us to predict certain physico-chemical properties. The first topological index, Wiener index, was published in 1947 [1], and the edge version of the Wiener index was proposed by Iranmanesh et al. in 2009 [2]. Because the important effects of the topological indices are proved in chemical research, more and more topological indices are studied, including the classical atom-bond connectivity index and the geometric arithmetic index.

A graph is nothing but a representation of any physical situation involving discrete objects and a relationship among them. A dominating set D for a graph G is a subset of V(G) such that any vertex not in D has at least one neighbor in D [3, 4]. The domination number γ(G) is the size of a minimum dominating set in G. Vizing‟s conjecture from 1968 states that for the Cartesian product of graphs G and H ,γ(G)γ(H) ≤ γ(G□H), and Clark and Suen (2000) proved that γ(G)γ(H)≤2γ(G□H). In this paper, we modify the approach of Clark and Suen to prove a variety of similar bounds related to total and paired domination,and also extend these bounds to then n-Cartesian product of graphs A^1 throughA^n.

II. LITERATURE REVIEW

B.Stjan et al., Vizing's conjecture from 1968 asserts that the domination number of the Cartesian product of two graphs is at least as large as the product of their domination numbers. In this paper we survey the approaches to this central conjecture from domination theory and give some new results along the way. For instance, several new properties of a minimal counterexample to the conjecture are obtained and a lower bound for the domination number is proved for products of claw-free graphs with arbitrary graphs. Open problems, questions and related conjectures are discussed throughout the paper.

W.Clark et al., let γ(G) denote the domination number of a graph G and let G H denote the Cartesian product of graphs G and H. We prove that γ(G)γ(H) ≤ 2γ(GH) for all simple graphs G and H.

P. T. Ho. et al., A total dominating set of a graph G with no isolated vertices is a subset S of the vertex set such that every vertex of G is adjacent to a vertex in S. The total domination number of G is the minimum cardinality of a total dominating set of G. In this paper, we study the total domination number of middle graphs. Indeed, we obtain tight bounds for this number in terms of the order of the graph G. We also compute the total domination number of the middle graph of some known families of graphs explicitly. Moreover, some Nordhaus-Gaddum-like relations are presented for the total domination number of middle graphs.

X. M. Hou et al., the most famous open problem involving domination in graphs is Vizing's conjecture which states the domination number of the Cartesian product of any two graphs is at least as large as the product of their domination

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numbers. We investigate a similar problem for paired-domination, and obtain a lower bound in terms of product of domination number of one factor and 3-packing of the other factor. Some results are obtained by applying a new graph invariant called rainbow domination.

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Cockayne E J et al., a set D of vertices of a finite, undirected graph G = (V, E) is a total dominating set if every vertex of V is adjacent to some vertex of D. In this paper we initiate the study of total dominating sets in graphs and, in particular, obtain results concerning the total domination number of G (the smallest number of vertices in a total dominating set) and the total domatic number of G (the largest order of a partition of G into total dominating sets).

Bert Hartnell et al., in this chapter, we study Vizing's Conjecture from 1968 which asserts that the domination number of the Cartesian product of two graphs is at least as large as the product of their domination numbers. The conjecture was first posed by Vizing as a question in 1963. Vizing's Conjecture is considered by many to be the main open problem in the area of domination in graphs. We also present Vizing-like conjectures for the total domination number, the independent domination number, the independence number, the upper domination number, and the upper total domination number in Cartesian products of graphs.

V.G. Vizing et al., we prove uniqueness of decomposition of a finite metric space into a product of metric spaces for a wide class of product operations. In particular, this gives the positive answer to the long-standing question of S. Ulam: 'If U × U ' V × V with U, V compact metric spaces, will then U and V be isometric?' in the case of finite metric spaces. In the proof we use uniqueness of cartesian decomposition of connected graphs; a known fact to which we give a new proof which is shorter and more transparent than existing ones.

HOU Xinmin et al., the Cartesian product of graphs is one of the most fundamental graph operations and has been studied since its introduction in the 1950's. Many important classes of graphs such as hypercubes, Hamming graphs, and prisms, are Cartesian products. Even more important is the fact that Cartesian products serve as natural hosts for different embeddings in metric graph theory. A prominent example is the canonical metric representation of an arbitrary graph due to Graham and Winkler [4], see also [3, 5]: every graphs has a unique (irredundant) isometric of , and is a subset of . A graph is called -antimagic if for each subset of with , there is an edge labeling with labels in such that the sums of the labels assigned to edges incident to distinct vertices are different. The main result of this paper is that the Cartesian products of complete graphs (except) and cycles are -antimagic.

Gao, W. et al., as the concepts in theoretical and applied areas, a graph is represented by a collection of connecting points and lines, who can be separately called vertices and edges. Suppose e is an edge of G, which connects the vertices u and v, then we denote and state that "u and v are adjacent". We can see that there is a path between every pair of vertices existing in a connected graph described above. Then the length of a shortest path between u and v in G can be determined by the distance $d(u, v)$ of two vertices u and v. A simple graph is defined as an unweighted, undirected graph who has no loops and multiple edges attached. Every individual number used to characterize some properties of a graph is known as a topological index of a related (molecular) graph. It's apparent that the numbers of vertices and edges are topological variants.

Vukicevic, D. et al., research on the topological indices based on end-vertex degrees of edges has been intensively rising recently. Randić index, one of the best-known topological indices in chemical graph theory, is belonging to this class of indices. In this paper, we introduce a novel topological index based on the end-vertex degrees of edges and its basic features are presented here. We named it as geometrical-arithmetic index (GA).

III. METHODOLOGY

We begin by introducing some notation which will be utilized throughout the proofs in this section. Given $S \subseteq V(G \square H)$, the **projection of S onto graphs G and H** is defined as $\Phi_G(S) = \{ g \in V(G) \mid \exists\, h \in V(H) \text{ with } gh \in S \}$, $\Phi_H(S) = \{ h \in V(H) \mid \exists\, g \in V(G) \text{ with } gh \in S\}$.In the case of the **n-product graph** $A^1 \square \ldots . \square A^n$, we project a set of

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vertices in V(A^1□….□A^n) down to a particular graph A^i. Therefore, given S ⊆ V(A^1□….□A^n), we define Φ ^{i}(S) = { a ∈ V(A^i) | ∃ A
$u^1 \cdots u^n$ ∈ S with a = u^i }.

For gh ∈ V(G□H), the **G-neighborhood** and **H-neighborhood** of gh are defined as follows:

$N_{\underline{G}\square H}$ (gh) = {g h ∈ V(G□H) | g ∈N_G(g)}, ′ ′

$N_{G\square \underline{H}}$ (gh) = {gh′ ∈ V(G□H) | h ∈N_H(h)}. Thus, $N_{\underline{G}\square H}$ (gh) and $N_{G\square \underline{H}}$ (gh) are both subsets of V(G□H). Additionally, E(G□H) can be partitioned into two sets, **G-edges** and **H-edges**, where G-edges = {(gh, g h) ∈ E(G□H) | h ∈ V(H) and (g, g)
∈ E(G)}, H-edges = {(gh, gh′) ∈ E(G□H) | g ∈ V(G)and (h, h) ∈ E(H)}.

′ ′ ′

In the case of the n-product graph A^1□….□A^n, we identify the **i-neighborhood** of a particular vertex, and partition the set of edges E(A^1□….□A^n) into n sets. Thus, we define E_i to be E_i = { (… , …) | (u^i,v^i) ∈ E(A^i), and u_j = v_j, for all other indices j ≠ i}, And for a vertex u ∈ V(A^1□….□A^n), we define $N_{\square A^i}$(u) = { v ∈ V(A^1□….□A^n) | v and u are connected by E_i-edge}. u^1 u^n v^1 v^n u_j

Finally, we need two elementary propositions about matrices that will be utilized throughout the proofs.

Domination in graphs has applications to several fields. Domination arises in facility location problems, where the number of facilities (e.g., hospitals, fire stations) is fixed and one attempts to minimize the distance that a person needs to travel to get to the closest facility. A similar problem occurs when the maximum distance to a facility is fixed and one attempts to minimize the number of facilities necessary so that everyone is serviced. Concepts from domination also appear in problems involving finding sets of representatives, in monitoring communication or electrical networks, and in land surveying (e.g., minimizing the number of places a surveyor must stand in order to take height measurements for an entire region).

SCHOOL BUS ROUTING

Most school in the country provide school buses for transporting children to and from school Most also operate under certain rules, one of which usually states that no child shall have to walk farther than, say one quarter km to a bus pickup point. Thus, they must construct a route for each bus that gets within one quarter km of every child in its assigned area. No bus ride can take more than some specified number of minutes, and Limits on the number of children that a bus can carry at any one time. Let us say that the following figure represents a street map of part of a city, where each edge represents one pick up block. The school is located at the large vertex. Let us assume that the school has decided that no child shall have to walk more than two blocks in order to be picked up by a school bus. Construct a route for a school bus that leaves the school, gets within two blocks of every child and returns to the school.

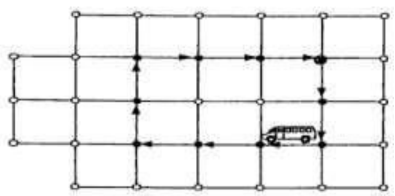

Figure 1: School Bus Routing

Computer Communication Networks

Consider a computer network modeled by a graph G = (V, E), for which vertices represents computers and edges represent direct links between pairs of computers. Let the vertices in following figure represent an array, or network, of 16 computers, or processors. Each processor to which it is directly connected. Assume that from time to time we need to collect information from all processors. We do this by having each processor route its information to one of a small set of collecting processors (a dominating set). Since this must be done relatively fast, we cannot route this information over too long a path. Thus we identify a small set of processors which are close to all other processors. Let us say that we will tolerate at most a two unit delay between the time a processor sends its information and the time it arrives at a nearby

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collector. In this case we seek a distance-2 dominating set among the set of all processors. The two shaded vertices form a distance- dominating set in the hypercube network infollowing figure

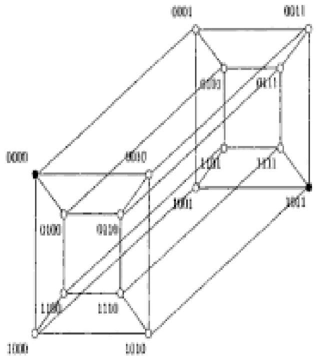

Figure -2 : Computer Communication Networks

Radio Station

Suppose that we have a collection of small villages in a remote part of the world. We would like to locate radio stations in some of these villages so that messages can be broadcast to all of the villages in the region. Since each radio station has a limited broadcasting range, we must use several stations to reach all villages. But since radio stations are costly, we want to locate as few as possible which can reach all other villages.

Let each village be represented by a vertex. An edge between two villages is labeled with the distance, say in kilometers, between the two villages

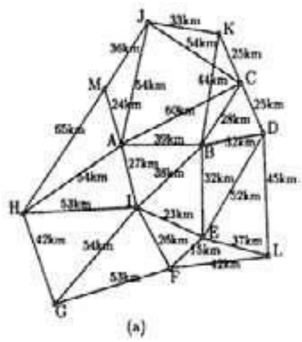

Figure-3: Radio Station

Let us assume that a radio station has a broadcast range of fifty kilometers. What is the least number of stations in a set which dominates (within distance 50) all other vertices in this graph? A set (B, F, H,J} of cardinality four is indicated in the following figure(b).

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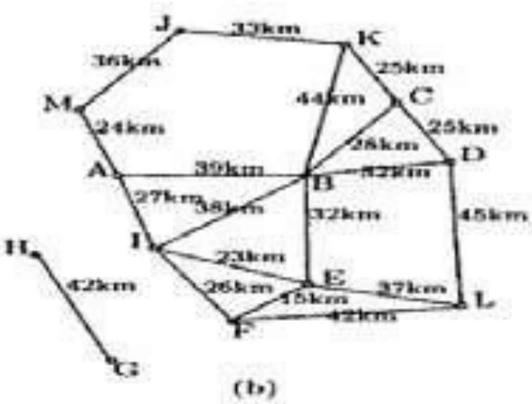

Figure-4: Radio Station

Here we have assumed that a radio station has a broadcast range of only fifty kilometers, we can essentially remove all edges in the graph, which represent a distance of more than fifty kilometers.

IV. CONCLUSION

This paper "Total And Paired Domination of Cartesian Product Graphs", can make an in-depth study in Total domination and Paired domination using Cartesian product graphs. We discussed the various applications of graph theory and dealt with the total and paired domination of Cartesian product graphs. We also dealt with the paired domination of Cartesian product graphs and the real life applications of domination in graphs.

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7.54

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