e-ISSN:2582-7219

INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY RESEARCH IN SCIENCE, ENGINEERING AND TECHNOLOGY

Volume 7, Issue 10, October 2024

INTERNATIONAL STANDARD SERIAL NUMBER INDIA

Impact Factor: 7.521

6381 907 438 6381 907 438 ijmrset@gmail.com www.ijmrset.com

ISSN: 2582-7219 | www.ijmrset.com | Impact Factor: 7.521| ESTD Year: 2018|

International Journal of Multidisciplinary Research in Science, Engineering and Technology (IJMRSET)

(A Monthly, Peer Reviewed, Refereed, Scholarly Indexed, Open Access Journal)

(S, d) Magic Labeling of Subdivision on Some Graphs

Dr P. Sumathi [1], P. Mala[2]

Department of Mathematics, C. Kandaswami College for Men, Anna Nagar, Chennai, India[1]

Department of Mathematics, St Thomas College of Arts and Science, Koyambedu, Chennai, India[2]

ABSTRACT: Let G (p, q) be a connected, undirected, simple and non-trivial graph with p vertices and q edges. Let f be an injective function f: V(G) → {s , $s+d, s+2d,$..s + $(q+1)$ $d\}$ and g be an injective function g: E(G) → $\{d, 2d, 3d$...$2(q-1)$ $d\}$. Then the function f is said to be (s, d) magic labeling if $(u) + g(uv) + f(v)$ is a constant, for all $u, v \in V(G)$ and $uv \in E(G)$. A graph G is called (S, d) magic graph if it admits (S, d) magic labeling.

KEYWORDS: Subdivision of Open ladder graph S (O(L_n)), Open triangular ladder S(O(TL_n)), Slanting ladder S(S$L_{n,}$) , Circular ladder $S(CL_n)$,Mobius ladder$S(M_n)$

I. INTRODUCTION

[4] In graph theory, subdivision is a critical concept that simplifies the analysis of complex graphs by transforming them into easier or simpler forms, enabling a clearer calculation of various graph properties. Subdivision graphs play an important role in studying both mathematical and chemical properties of the objects represented by these graphs, such as molecules or networks.

[3]. In 2001, Sethuraman and Selvaraju introduced a graph operation called the super subdivision of a graph, denoted as SSD(G). The operation creates a new graph from an original graph G by transforming each edge in a specific manner using a complete bipartite graph.

If G admits (s, d) Magic labeling, then G is called as (s, d) Magic graph. In this paper, a new concept of (s, d) Magic labeling has been introduced for some graphs.

[5] Let $G\ (p, q)$ be a simple, non-trivial, connected, undirected graph with p vertices and q edges. Consider the following: $f: (G) \rightarrow \{s, s + d, s + 2d... s + (q + 1)\ d\}$ and $g: E(G) \rightarrow \{d, 2d, 3d \ldots 2(q - 1)\ d\}$ be an injective function. Then, for any $u, v \in (G)$ and $uv \in E(G)$, $f(u) + g(uv) + f(v)$ is a constant, and the function f is said to be (S, d) magic labeling. If a graph G admits (S, d) magic labeling, then it is referred to as a (S, d) magic graph.

II. DEFINITIONS

Definition 2.1

The graph obtained by subdividing each edge of a graph G is called the subdivision graph of G.

Definition 2.2

Two paths of length $n-1$ with $(G) = \{u_i v_i : 1 \leq i \leq n$ and

$(G) = \{u_i u_{i+1}, v_i v_{i+1} :\ 1 \leq i \leq n-1\} \cup \{u_i v_i : 2 \leq i \leq n-1\}$ form an Open ladder.

Definition 2.3

The slanting ladder is a graph that consists of two copies of Pn with vertex set $\{u_i : 1\ \leq\ i\ \leq\ n\}$ $\cup$ $\{v_i : 1\ \leq\ i\ \leq\ n\}$ and edge set is generated by linking u_i and v_{i+1}, $1 \leq i \leq n\ -\ 1$.

Definition 2.4

The graph TLn n≥ 2 is formed by adding the edges $u_i v_{i+1}$: $1\ \leq\ i\ \leq\ n\ -\ 1$, to the ladder, where Ln is the graph P_2 × Pn.

Definition 2.5

By adding the edges $u_i v_{i+1}\ for\ 1 \leq i \leq n-1$, a triangular ladder is modified in to open triangular ladder and it is denoted as O (TL_n)

Definition 2.6

Circular ladder graph is a simple graph obtained by using Cartesian product of cycle graph

ISSN: 2582-7219 | www.ijmrset.com | Impact Factor: 7.521| ESTD Year: 2018|

International Journal of Multidisciplinary Research in Science, Engineering and Technology (IJMRSET)

(A Monthly, Peer Reviewed, Refereed, Scholarly Indexed, Open Access Journal)

C_n with n vertices and path graph P_2, and is denoted by CL_n. (i.e. $C_n \times P_2 = CL_n$). This is isomorphic to the graph obtained by linking the end vertices of the ladder by two new edges in cyclic form.

Definition 2.7

A Mobius ladder graph M_n is a graph obtained from the ladder $P_n \times P_2$ by linking the opposite end points of the two copies of P_n.

III. MAIN RESULT

Theorem 3.1 Subdivision of any Open ladder graph $(OL_n)$for all η is odd is (S, d) magic labeling.

Proof: Let G = (OL_n) be a graph obtain by subdividing all the edges of $S(OL_n)$

Here we consider a following case

Case (a)

Let G be a graph obtained by subdividing each edge $u_i v_i, 2 \leq i \leq \eta - 1$ of L_n

Let w_i, $1 \leq i \leq \eta - 2$ be the vertices which subdivide $u_i\ v_i$

Define the function f from the vertex set to $\{s, s+d, s+2d \ldots . s+(q+1)\}$,

$g: (G) \rightarrow \{d, 2d, 3d \ldots .2(q-1)\ d\}$

Labeling of vertices			
Value of i	$f(u_{i+1})$	$f(w_{i+1})$	$f(v_{i+1})$
$0 \leq i \leq \eta - 1$	$S + id$	-	$s + (2(\eta - 1) + i)d$
$0 \leq i \leq \eta - 3$	-	$s + (\eta + i)d$	−

Table 1 Labeling of vertices of (OL_n)

Labeling of Edges				
Value of i	$g(u_i u_{i+1})$	$g(v_i v_{i+1})$	$g(u_{i+1} w_i)$	$g(w_i v_{i+1})$
$1 \leq i \leq \eta - 1$	$2s + 2(q-1)\ d - (f(u_i) + f(u_{i+1}))$	$2s + 2(q-1)\ d - f(v_i) + f(v_{i+1}))$	-	-
$1 \leq i \leq \eta$-2	-	-	$2s + 2(q-1)d - (f(u_{i+1}) + f(w_i))$	$2s + 2(q-1)\ d - (f(w_i) + f(v_{i+1}))$

Table 2 Labeling of Edges of (OL_n)

Example 3.1.1: Subdivision of Open ladder graph (OL_7)

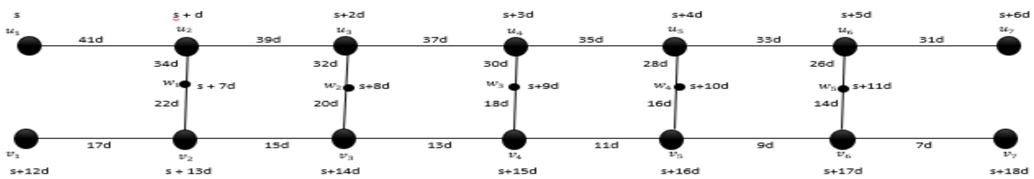

Fig. 1 Subdivision of Open ladder graph (OL_7)

© 2024 IJMRSET | Volume 7, Issue 10, October 2024| DOI: 10.15680/IJMRSET.2024.0710025

ISSN: 2582-7219 | www.ijmrset.com | Impact Factor: 7.521| ESTD Year: 2018|

International Journal of Multidisciplinary Research in Science, Engineering and Technology (IJMRSET)

(A Monthly, Peer Reviewed, Refereed, Scholarly Indexed, Open Access Journal)

Case (b)

Let G be a graph obtained by subdividing each edge u_iu_{i+1} and v_iv_{i+1} of L_n

Let w_{i+1}, $0 \leq i \leq (\eta - 2)$ be the vertices which subdivide u_iu_{i+1} and y_{i+1}, $0 \leq i \leq (\eta - 1)$ be the vertices which subdivide v_iv_{i+1} of L_n

Define the function f from the vertex set to $\{\{s, s + d, s + 2d \ldots . s + (q + 1)\}$,

$g: (G) \rightarrow \{d, 2d, 3d \ldots . .2(q - 1)\, d\}$

Labeling of vertices				
Value of i	$f(u_{i+1})$	$f(w_{i+1})$	$f(v_{i+1})$	$f(y_{i+1})$
$0 \leq i \leq \eta - 1$	$S + 2id$	-	$s + 2(\eta + i)d$	$s + 2(\eta + i)\, d - id$
$0 \leq i \leq (\eta - 2)$	-	$s + (2i + 1)d$	−	

Table 3 Labeling of vertices of (OL_n)

Labeling of Edges					
Value of i	$g(u_iw_i)$	$g(w_iu_{i+1})$	$g(v_iy_i)$	$g(y_iv_{i+1})$	$g(u_iv_i)$
$1 \leq i \leq \eta - 1$	$2s + 2(q - 1)\, d - f(u_i) + f(w_i))$	$2s + 2(q - 1)\, d - f(w_i) + f(u_{i+1}))$	$2s + 2(q - 1)d - (f(v_i) + f(y_i))$	$2s + 2(q - 1)d - ((f(y_i) + f(v_{i+1}))$	
$2 \leq i \leq \eta$-1	-	-	-	-	$2s + 2(q - 1)d - (f(u_i) + f(v_i))$

Table 4 Labeling of Edges of (OL_n)

Example 3.1.2: Subdivision of Open ladder graph (OL_5)

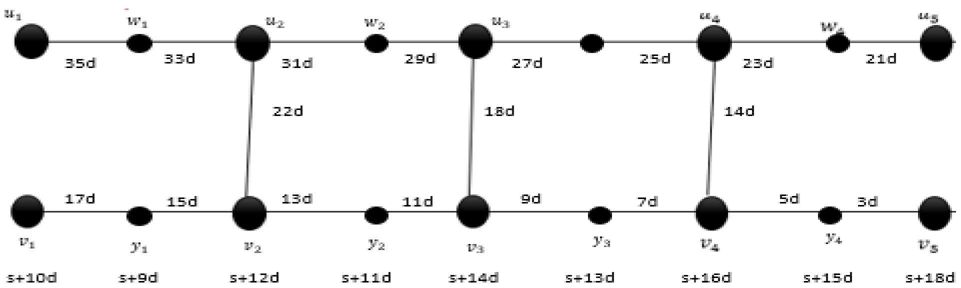

Fig. 2 Subdivision of Open ladder graph (OL_5)

© 2024 IJMRSET | Volume 7, Issue 10, October 2024| DOI: 10.15680/IJMRSET.2024.0710025

ISSN: 2582-7219 | www.ijmrset.com | Impact Factor: 7.521| ESTD Year: 2018|

International Journal of Multidisciplinary Research in Science, Engineering and Technology (IJMRSET)

(A Monthly, Peer Reviewed, Refereed, Scholarly Indexed, Open Access Journal)

Case (c)

Let G be a graph obtained by subdividing each edge u_iu_{i+1}, v_iv_{i+1} and u_iv_i of L_n

Let w_{i+1} ;$0 \leq i \leq (\eta - 1)$ be the vertices which subdivide u_iu_{i+1}, y_{i+1}; $0 \leq i \leq (\eta - 2)$ be the vertices which subdivide $v_iv_{i+1}$and x_i; $0 \leq i \leq (\eta - 2)$ be the vertices which subdivide u_iv_i; $2 \leq i \leq \eta - 1$ of L_n

Define the function f from the vertex set to $\{\{s, s + d, s + 2d \ldots . s + (q + 1)\}$,

$g: (G) \rightarrow \{d, 2d, 3d \ldots . .2(q - 1)\ d\}$

Labeling of vertices					
$f(v_\eta) = v_{n-1} + d$					
Value of i	$f(u_{i+1})$	$f(w_{i+1})$	$f(v_{i+1})$	$f(y_{i+1})$	$f(x_i)$
$0 \leq i \leq \eta - 1$	$S + 2id$	$s + (2i + 1)d$	–	–	–
$0 \leq i \leq \eta - 2$	–	–	$s + 2(\eta + i)d$	$s + 2(\eta + i)\ d - d$	–
$1 \leq i \leq \eta - 2$	–	–	–	–	$v_{n-1} + 2id$

Table 5 Labeling of vertices of (OL_n)

Labeling of Edges						
Value of i	$g(u_iw_i)$	$g(w_iu_{i+1})$	$g(v_iy_i)$	$g(y_iv_{i+1})$	$g(x_iu_{i+1})$	$g(x_iv_{i+1})$
$1 \leq i \leq \eta - 1$	$2s + 2(q - 1)\ d - f(u_i) + f(w_i))$	$2s + 2(q - 1)d - (f(w_i) + f(u_{i+1}))$	$2s + 2(q - 1)\ d - f(v_i) + f(y_i))$	$2s + 2(q - 1)d - ((f(y_i)) + f(v_{i+1}))$	–	–
$1 \leq i \leq \eta$-2	–	–	–	–	$2s + 2(q - 1)\ d - (f(x_i) + f(u_{i+1}))$	$2s + 2(q - 1)d - (f(x_i) + f(v_{i+1}))$

Table 6 Labeling of Edges of (OL_n)

Thus, (OL_n) admits (s, d) magic labeling

Example 3.1.3: Subdivision of Open ladder graph (OL_7)

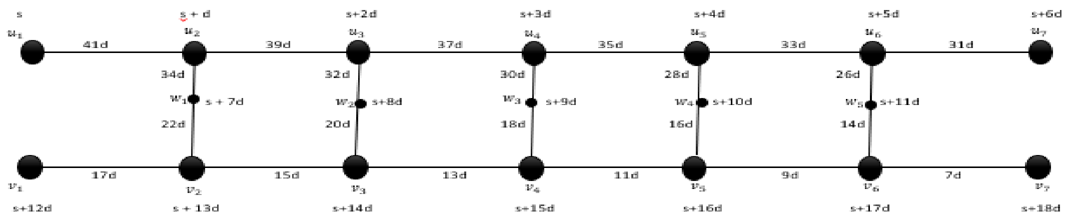

Fig. 3 Subdivision of Open ladder graph (OL_7)

© 2024 IJMRSET | Volume 7, Issue 10, October 2024| DOI: 10.15680/IJMRSET.2024.0710025

ISSN: 2582-7219 | www.ijmrset.com | Impact Factor: 7.521| ESTD Year: 2018|

International Journal of Multidisciplinary Research in Science, Engineering and Technology (IJMRSET)

(A Monthly, Peer Reviewed, Refereed, Scholarly Indexed, Open Access Journal)

Theorem 3.2 Subdivision of a Slanting ladder graph $(SL_n)$for η is even is (S, d) magic labeling.
Proof: Let G = (SL_n) be a graph obtain by subdividing all the edges of $S(SL_n)$
Here we consider a following case
Case (a)
Let G be a graph obtained by subdividing each edge $u_i v_{i+1}$, of (SL_n)
Let w_i, $1 \leq i \leq \eta - 1$ be the vertices which subdivide $u_i \; v_{i+1}$
Define the function f from the vertex set to $\{s, s+d, s+2d \ldots . s+(q+1)\}$,
$g\colon(G) \rightarrow \{d, 2d, 3d \ldots . .2(q-1)\, d\}$

Labeling of vertices		
Value of i	$f(u_{i+1})$	$f(v_i)$
$0 \leq i \leq \eta - 1$	$S + id$	–
$1 \leq i \leq \eta$	-	$w_\eta + id$

Table 7 Labeling of vertices of (SL_n)

Labeling of Edges				
Value of i	$g(u_i u_{i+1})$	$g(v_i v_{i+1})$	$g(u_i w_i)$	$g(w_i v_{i+1})$
$1 \leq i \leq \eta - 1$	$2s + 2(q-1)\,d - f(u_i) + f(u_{i+1}))$	$2s + 2(q-1)\,d - (f(v_i) + f(v_{i+1}))$	$2s + 2(q-1)\,d - (f\,(u_i) + f(w_i)$	$2s + 2(q-1)\,d - f(w_i) + f(v_{i+1}))$

Table 8 Labeling of Edges of (SL_n)

Example 3.2.1: Subdivision of slanting ladder graph (SL_6)

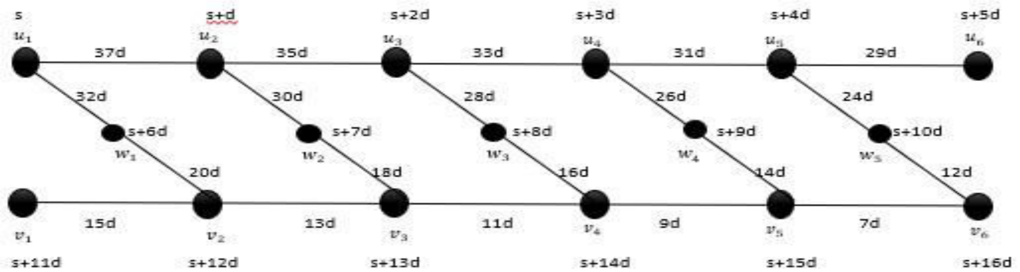

Fig. 4 Subdivision of slanting ladder graph (SL_6)

ISSN: 2582-7219 | www.ijmrset.com | Impact Factor: 7.521| ESTD Year: 2018|

International Journal of Multidisciplinary Research in Science, Engineering and Technology (IJMRSET)

(A Monthly, Peer Reviewed, Refereed, Scholarly Indexed, Open Access Journal)

Case (b)

Let G be a graph obtained by subdividing each edge $u_iu_{i+1}$and v_iv_{i+1} of (SL_n)

Let w_i, $1 \leq \ i \leq \eta - 1$ be the vertices which subdivide $u_i\ u_{i+1}$ and y_i, $1 \leq \ i \leq \eta - 1$ be the vertices which subdivide v_i

Define the function f from the vertex set to $\{s, s+d, s+2d \ldots. s+(q+1)\}$,

$g: (G) \to \{d, 2d, 3d \ldots..2(q-1)\ d\}$

Labeling of vertices				
Value of i	$f(u_{i+1})$	$f(v_{i+1})$	$f(w_{i+1})$	$f(y_{i+1})$
$0 \leq i \leq \eta - 1$	$S + 4id$	$s + (4i+1)d$	$s + 2(1+2i)d$	$s + (4i+3)d$

Table 9 Labeling of vertices of (SL_n)

	Labeling of Edges				
Value of i	$g(u_iv_{i+1})$	$g(y_iv_{i+1})$	$g(v_iy_i)$	$g(u_iw_i)$	$g(w_iu_{i+1})$
$1 \leq i \leq \eta - 1$	$2s + 2(q-1)\ d - (f(u_i) + f(v_{i+1}))$	$2s + 2(q-1)\ d - (f(y_i) + f(v_{i+1}))$	$2s + 2(q-1)d - (f(v_i) + f(y_i))$	$2s + 2(q-1)\ d - f(u_i) + f(w_i))$	$2s + 2(q-1)\ d - (f(w_i) + f(u_{i+1}))$

Table 10 Labeling of Edges of (SL_n)

Example 3.2.2: Subdivision of slanting ladder graph (SL_6)

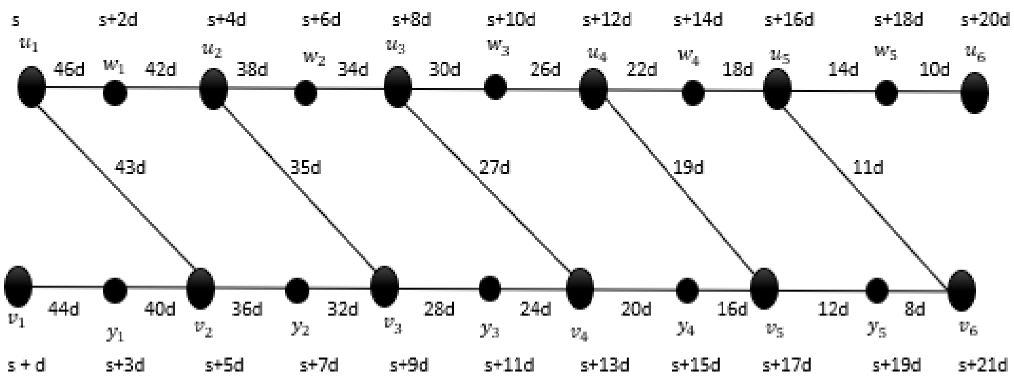

Fig. 5 Subdivision of slanting ladder graph (SL_6)

© 2024 IJMRSET | Volume 7, Issue 10, October 2024| DOI: 10.15680/IJMRSET.2024.0710025

ISSN: 2582-7219 | www.ijmrset.com | Impact Factor: 7.521| ESTD Year: 2018|

International Journal of Multidisciplinary Research in Science, Engineering and Technology (IJMRSET)

(A Monthly, Peer Reviewed, Refereed, Scholarly Indexed, Open Access Journal)

Case (c)

Let G be a graph obtained by subdividing each edge u_iu_{i+1}, v_iv_{i+1} and $u_iv_i$of (SL_n)

Let w_i, $1 \leq i \leq \eta - 1$ be the vertices which subdivide $u_i\ u_{i+1}$, y_i $1 \leq i \leq \eta - 1$ be the vertices which subdivide v_i and x_i; $1 \leq i \leq \eta - 1$ be the vertices which subdivide $u_i\ v_{i+1}$ Define the function f from the vertex set to $\{s, s + d, s + 2d \ldots . s + (q + 1)\}$,

$g: (G) \rightarrow \{d, 2d, 3d \ldots . .2(q - 1)\ d\}$

Labeling of vertices					
Value of i	$f(u_{i+1})$	$f(w_{i+1})$	$f(v_i)$	$f(y_i)$	$f(x_i)$
$0 \leq i \leq \eta - 1$	$S + 2id$	-	-	$s + 2(\eta - 1 + i)d$	−
$0 \leq i \leq \eta - 2$	--	$s + (2i + 1)d$	−	−	$u_\eta + (2i + 1)d$
$1 \leq i \leq \eta$	−	−	$s + (x_{\eta-1} + i)d$	−	−

Table 11 Labeling of vertices of (SL_n)

Labeling of Edges						
Value of i	$g(u_iw_i)$	$g(w_iu_{i+1})$	$g(v_iy_i)$	$g(y_iv_{i+1})$	$g(u_ix_i)$	$g(x_iv_{i+1})$
$1 \leq i \leq \eta - 1$	$2s + 2(q - 1)d - f(u_i) + f(w_i))$	$2s + 2(q - 1)d - f(w_i) + f(u_{i+1}))$	$2s + 2(q - 1)\ d - (f(v_i) + f(y_i))$	$2s + 2(q - 1)d - ((f\ (y_i) + f(v_{i+1}))$	$2s + 2(q - 1)\ d - (f(u_i) + f(x_i))$	$2s + 2(q - 1)d - ((f\ (x_i) + f(v_{i+1}))$

Table 12 Labeling of Edges of (SL_n)

Thus, (SL_n) admits (S, d) magic labeling

Example 3.2.3: Subdivision of slanting ladder graph (SL_6)

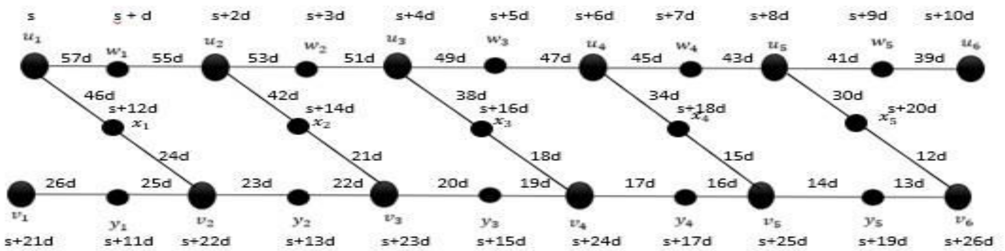

Fig. 6 Subdivision of slanting ladder graph (SL_6)

© 2024 IJMRSET | Volume 7, Issue 10, October 2024| DOI: 10.15680/IJMRSET.2024.0710025
ISSN: 2582-7219 | www.ijmrset.com | Impact Factor: 7.521| ESTD Year: 2018|
International Journal of Multidisciplinary Research in Science, Engineering and Technology (IJMRSET)
(A Monthly, Peer Reviewed, Refereed, Scholarly Indexed, Open Access Journal)

Theorem 3.3 Subdivision of an Open triangular ladder graph $(OTL_n)$for all η is (S, d) magic labeling.
Proof: Let G = (OTL_n) be a graph obtain by subdividing all the edges of $S(OTL_n)$
Here we consider a following case
Case (a)
Let G be a graph obtained by subdividing each edge $u_iu_{i+1}$and v_iv_{i+1} of (OTL_n)
Let w_i, $1 \leq i \leq \eta - 1$ be the vertices which subdivide $u_i\, u_{i+1}$and y_i, $1 \leq i \leq \eta - 1$ be the vertices which subdivide $v_i\, v_{i+1}$
Define the function f from the vertex set to $\{s, s + d, s + 2d \ldots . s + (q + 1)\}$, $g: (G) \rightarrow \{d, 2d, 3d \ldots .. 2(q - 1)\, d\}$

Labeling of vertices				
Value of i	$f(u_{i+1})$	$f(v_{i+1})$	$f(w_{i+1})$	$f(y_{i+1})$
$0 \leq i \leq \eta - 1$	$S + 4id$	$s + (4i + 1)d$	$s + 2(1 + 2i)d$	$s + (4i + 3)d$

Table 13 Labeling of vertices of (OTL_n)

Labeling of Edges						
Value of i	$g(u_iv_{i+1})$	$g(y_iv_{i+1})$	$g(u_iw_i)$	$g(w_iu_{i+1})$	$g(v_iy_i)$	$g(v_iu_i)$
$1 \leq i \leq \eta - 1$	$2s + 2(q - 1)d - (f(u_i)) + f(v_{i+1}))$	$2s + 2(q - 1)\, d - (f(y_i) + f(v_{i+1}))$	$2s + 2(q - 1)\, d - (f(u_i) + f(wi))$	$2s + 2(q - 1)\, d - (f(w_i) + f(u_{i+1}))$	$2s + 2(q - 1)\, d - (f(v_i) + f(y_i))$	−
$2 \leq i \leq \eta - 1$	−	-	-	-	-	$2s + 2(q - 1)\, d - (f(v_i) + f(u_i))$

Table 14 Labeling of edges of (OTL_n)

Example 3.3.1 Subdivision of an Open triangular ladder graph (OTL_6)

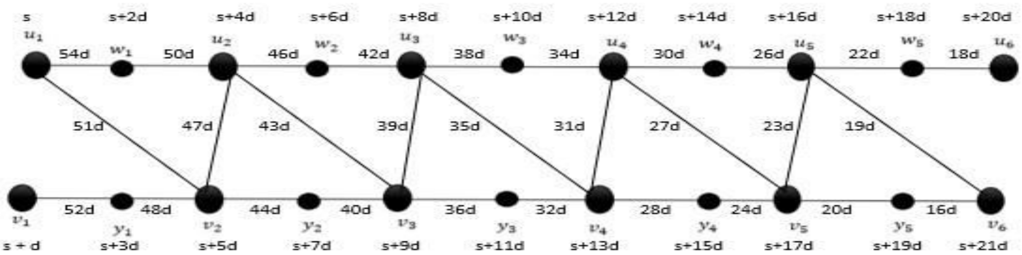

Fig. 7 Subdivision of an Open triangular ladder graph (OTL_6)

ISSN: 2582-7219 | www.ijmrset.com | Impact Factor: 7.521| ESTD Year: 2018|

International Journal of Multidisciplinary Research in Science, Engineering and Technology (IJMRSET)

(A Monthly, Peer Reviewed, Refereed, Scholarly Indexed, Open Access Journal)

Case (b)
Let G be a graph obtained by subdividing each edge $u_i v_{i+1}$ and $v_i u_i$ of (OTL_n)
Let x_i, $1 \leq i \leq 2\eta - 3$ be the vertices which subdivide $u_i v_{i+1}$ and $u_i\ v_i$;2≤i≤ η-1 Define the function f from the vertex set to $\{s, s+d, s+2d \ldots . s+(q+1)\}$, $g:(G) \rightarrow \{d, 2d, 3d \ldots ..2(q-1)\, d\}$

Value of i	$f(u_{i+1})$	$f(v_{i+1})$	$f(x_{i+1})$
$0 \leq i \leq \eta - 1$	$S + 4(i+1)d$	$s + 2(2i+1)d$	–
$i = 0$	–	–	s
$1 \leq i \leq 2(\eta - 2)$	–	–	$s + 2(i-1)d + d$

Table 15 Labeling of vertices of (OTL_n)

Labeling of Edges						
Value of i	$g(u_i u_{i+1})$	$g(v_i v_{i+1})$	$g(u_{i+1} x_{2i+1})$	$g(u_{i+1} x_{2i})$	$g(v_{i+1} x_{2i+1})$	$g(v_{i+1} x_{2i})$
$1 \leq i \leq \eta - 1$	$2s + 2(q-1)\, d - (f(u_i) + f(u_{i+1}))$	$2s + 2(q-1)\, d - (f(v_i) + f(v_{i+1}))$	–	–	–	$2s + 2(q-1)\, d - (f(v_{i+1}) + f(x_{2i}))$
$0 \leq i \leq \eta - 2$	–	–	$2s + 2(q-1)\, d - (f(u_{i+1}) + f(x_{2i+1}))$	–	$2s + 2(q-1)\, d - (f(v_{i+1}) + f(x_{2i+1})$	–
$1 \leq i \leq \eta - 2$	–	–	–	$2s + 2(q-1)\, d - (f(u_{i+1}) + f(x_{2i}))$	–	–

Table 16 Labeling of Edges of (OTL_n)

Thus, (OTL_n) admits (S, d) magic labeling.
Example 3.3.2 Subdivision of an Open triangular ladder graph (OTL_6)

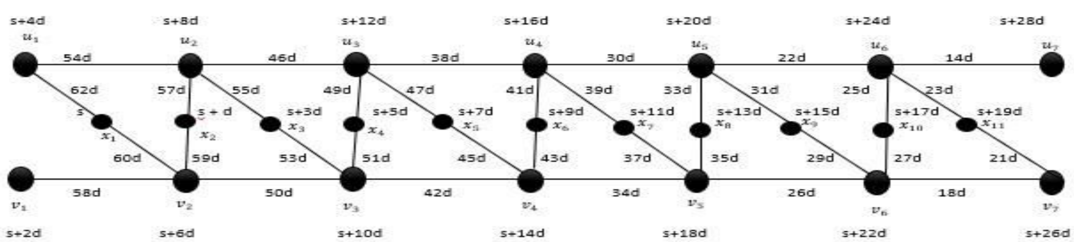

Fig. 8 Subdivision of an Open triangular ladder graph (OTL_6)

© 2024 IJMRSET | Volume 7, Issue 10, October 2024| DOI: 10.15680/IJMRSET.2024.0710025

ISSN: 2582-7219 | www.ijmrset.com | Impact Factor: 7.521| ESTD Year: 2018|

International Journal of Multidisciplinary Research in Science, Engineering and Technology (IJMRSET)

(A Monthly, Peer Reviewed, Refereed, Scholarly Indexed, Open Access Journal)

Theorem 3.4 A Subdivision of a circular ladder graph $(CL_n)$for η is odd is (S, d) magic labeling.
Proof: Let G = (CL_n) be a graph obtain by subdividing all the edges of $S(CL_n)$
$(S(L_n)) = \{u_i, v_i, r_i, s_i, t_i; 1 \leq i \leq \eta\}$ and $E(S(L_n)) = \{u_i r_i, r_i v_i, v_i s_i, u_i t_i;\ 1 \leq i \leq \eta\}$

Here $|(S(L_n))| = 5\eta$
$|(S(L_n))| = 6\eta$

Define the function f from the vertex set to $\{s, s+d, s+2d \ldots . s+(q+1)\}$, $g:(G) \rightarrow \{d, 2d, 3d \ldots .. 2(q-1)\ d\}$

Labeling of vertices					
Value of i	$f(u_{i+1})$	$f(v_i)$	$f(s_i)$	$f(r_i)$	$f(t_i)$
$0 \leq i \leq \eta - 1$	$s + id$	−	−	−	−
$1 \leq i \leq \eta - 1$	−	−	$s + (\eta + i)d$	−	$s + (v_\eta + i)d$
$1 \leq i \leq \eta$	−	$s + (r_\eta + i)$	−	$s + (s_\eta + i)d$	−

Table 17 Labeling of vertices of (CL_n)

Labeling of Edges						
Value of i	$g(u_i r_i)$	$g(v_i r_i)$	$g(u_i s_i)$	$g(v_i t_i)$	$g(s_i u_{i+1})$	$g(t_i v_{i+1})$
i = 1&η	−	−	$2s + 2(q-1)d\ (f(u_i) + f(s_\eta))$	$2s + 2(q-1)d - (f(v_i) + f(t_\eta))$	−	−
$1 \leq i \leq \eta$	$2s + 2(q-1)\ d - (f(u_i) + f(r_i))$	$2s + 2(q-1)\ d - (f(v_i) + f(r_i))$	$2s + 2(q-1)d-(f(ui) + f(si))$	$2s + 2(q-1)d - (f(v_i) + f(t_i))$	−	-−
$1 \leq i \leq \eta - 1$	−	−	−	−	$2s + 2(q-1)\ d - (f(s_i) + f(u_{i+1}))$	$2s + 2(q-1)\ d - (f(t_i) + f(v_{i+1}))$

Table 18 Labeling of Edges of (CL_n)

Thus, (CL_n) admits (S, d) magic labeling
Example 3.4.1 Subdivision of a circular ladder graph (CL_7)

ISSN: 2582-7219 | www.ijmrset.com | Impact Factor: 7.521| ESTD Year: 2018|

International Journal of Multidisciplinary Research in Science, Engineering and Technology (IJMRSET)

(A Monthly, Peer Reviewed, Refereed, Scholarly Indexed, Open Access Journal)

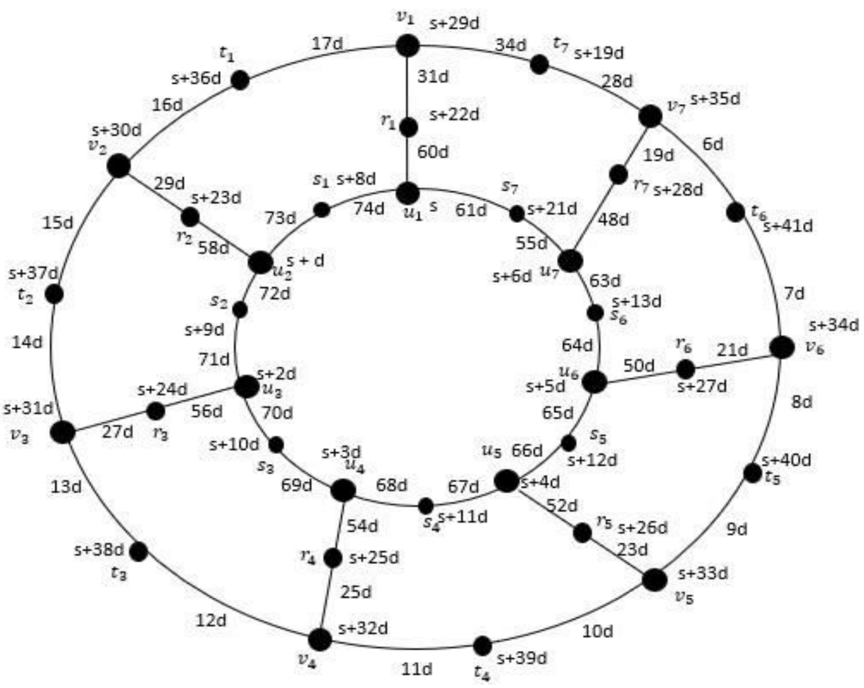

Fig. 9 Subdivision of a circular ladder graph (CL_7)

Theorem 3.5 A Subdivision of a Mobius ladder graph $(ML_n)$for η is odd is (S, d) magic labeling.
Proof: Let G $= (ML_n)$ be a graph obtain by subdividing all the edges of $S(ML_n)$
$(S(L_n)) = \{u_i, v_i, r_i, s_i, t_i; 1 \le i \le \eta\}$ and $E(S(L_n)) = \{u_i r_i, r_i v_i, v_i s_i u_i t_i\ ; 1 \le i \le \eta\}$

',

Define the function f from the vertex set to $\{s, s + d, s + 2d \ldots. s + (q + 1)\}$, $g{:}(G) \rightarrow \{d, 2d, 3d \ldots..2(q - 1)\, d\}$

Labeling of vertices					
$f(x) = s_{\eta-1} + d$					
$f(y) = s + 6\eta d$					
Value of i	$f(u_{i+1})$	$f(v_i)$	$f(s_i)$	$f(r_i)$	$f(t_i)$
$0 \le i \le \eta - 1$	$s + id$	−	−	−	−
$1 \le i \le \eta - 1$	−	−	$s + (\eta + i)d$	−	$s + (v_\eta + i)d$
$1 \le i \le \eta$	−	$s + (r_\eta + i)$	−	$s + (s_\eta + i)d$	−

Table 19 Labeling of vertices of $(ML_n$

ISSN: 2582-7219 | www.ijmrset.com | Impact Factor: 7.521| ESTD Year: 2018|

International Journal of Multidisciplinary Research in Science, Engineering and Technology (IJMRSET)

(A Monthly, Peer Reviewed, Refereed, Scholarly Indexed, Open Access Journal)

Labeling of Edges								
$g(yv_\eta)=2s+2(q-1)d-(f(y)+f(x_\eta))$								
$g(xu_\eta)=2s+2(q-1)d-(f(x)+f(u_\eta))$								
Value of i	$g(u_i r_i)$	$g(v_i r_i)$	$g(u_i s_i)$	$g(v_i t_i)$	$g(s_i u_{i+1})$	$g(t_i v_{i+1})$	$g(u_i y)$	$g(v_i x)$
$i=1$	–	–	–	–	–	–	$2s+2(q-1)d-(f(u_i)+f(y))$	$2s+2(q-1)d-(f(v_i)+f(x))$
$1\le i\le \eta$	$2s+2(q-1)d-(f(u_i)+f(r_i))$	$2s+2(q-1)d-(f(v_i)+f(r_i))$	–	–	–	–	–	–
$1\le i\le \eta -$	–	–	$2s+2(q-1)d-(f(u_i)+f(s_i))$	$-2s+2(q-1)d-(f(v_i)+f(t_i))$	$2s+2(q-1)d-(f(s_i)+f(u_{i+1}))$	$2s+2(q-1)d-(f(t_i)+f(v_{i+1}))$	–	–

Table 20 Labeling of vertices of (ML_n)

Thus, (ML_n) admits (S, d) magic labeling.
Example 3.5.1 Subdivision of a Mobius ladder graph (ML_7)

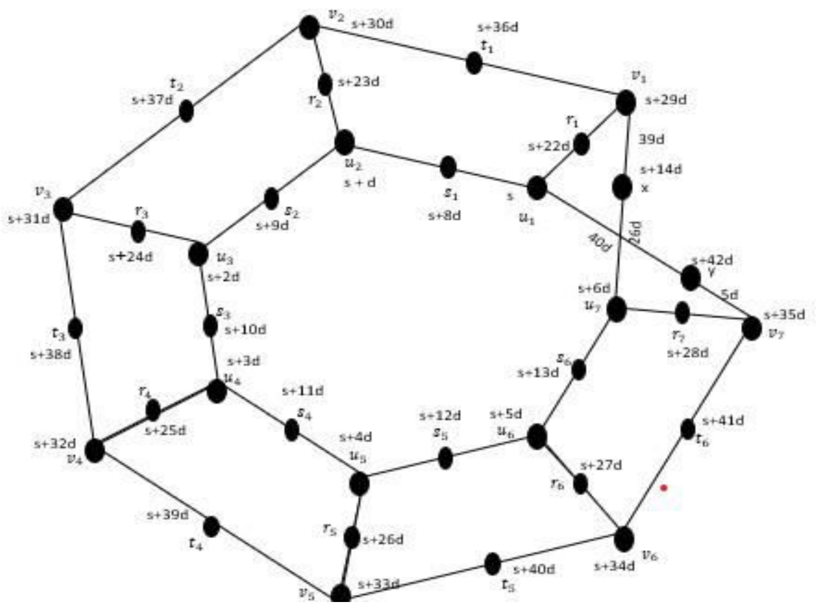

Fig. 10 Subdivision of a Mobius ladder graph (ML_7)

© 2024 IJMRSET | Volume 7, Issue 10, October 2024| DOI: 10.15680/IJMRSET.2024.0710025

ISSN: 2582-7219 | www.ijmrset.com | Impact Factor: 7.521| ESTD Year: 2018|

International Journal of Multidisciplinary Research in Science, Engineering and Technology (IJMRSET)

(A Monthly, Peer Reviewed, Refereed, Scholarly Indexed, Open Access Journal)

IV. CONCLUSIONS

In this study, a (S, d) Magic Labeling has been discovered for a few graphs such as Subdivision of Open ladder graph S (O (L_n)), Open triangular ladder S (O(TL_n)), Slanting ladder S(SL_n,) , Circular ladder $S(CL_n)$,Mobius ladder$S(M_n)$ .Future research will examine the (S, d) Magic labeling of additional graphs and some graph families.

REFERENCES

1. Gallian JA. A Dynamic Survey of Graph Labeling. The Electronic Journal of Combinatorics. 2022. Available from: http://www.combinatorics.org
2. Balamurugan, B.J., Thirusangu, K., Thomas, D.G.: k-Zumkeller labeling for twig graphs. Electron. Notes Discrete Math. 48, 119–126 (2015)
3. . Sethuraman, G., Selvaraju, P.: Gracefulness of arbitrary of super subdivision of graphs. Indian J. Pure Appl. Math. 32(7), 1059–1064 (2001). Natl. Acad. Sci. Lett. 26, 210–213 (2003)
4. Ascioglu, M., Cangul, I.: Narumi–Katayama index of the subdivision graphs. J. Taibah Univ. Sci. 12(4), 401–408 (2018)
5. Sumathi, P., & Mala, P. (2023). (s, d) Magic Labeling of some ladder graphs. In *E3S Web of Conferences* (Vol. 376, p. 01110). EDP Sciences.
6. Sumathi, P., & Kumar, J. S. (2022). Fuzzy Quotient-3 Cordial Labeling on Some Cycle Related Graphs. International Research Journal of Innovations in Engineering and Technology, 6(9), 49
7. Sandhya S. S., Somasundaram S., and Anusa S., Root square mean labeling of graphs, International Journal of Contemporary Mathematical Sciences. 2014; 9(14) :667- 676.
8. Wang T. M., Toroidal grids are anti-magic, Computing and Combinatorics, Lecture Notes in Compute. Sci., 3595, Springer, Berlin. 2005; 671 –679.

INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY RESEARCH

IN SCIENCE, ENGINEERING AND TECHNOLOGY

| Mobile No: +91-6381907438 | Whatsapp: +91-6381907438 | ijmrset@gmail.com |

www.ijmrset.com