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# HOW DO YOU CALCULATE DIFFERENTIAL n

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**ABSTRACT:** In Mathematics, a differential equation is an equation that contains one or more functions with its derivatives. The derivatives of the function define the rate of change of a function at a point. It is mainly used in fields such as physics, engineering, biology and so on. The primary purpose of the differential equation is the study of solutions that satisfy the equations and the properties of the solutions. Learn how to solve differential equations here.

One of the easiest ways to solve the differential equation is by using explicit formulas. In this article, let us discuss the definition, types, methods to solve the differential equation, order and degree of the differential equation, ordinary differential equations with real-word examples and a solved problem.

**KEYWORDS**-differential, calculate, n, equation, methods, order

## I. INTRODUCTION

A differential equation is an equation which contains one or more terms and the derivatives of one variable (i.e., dependent variable) with respect to the other variable (i.e., independent variable)

$$dy/dx = f(x)$$

Here “x” is an independent variable and “y” is a dependent variable

For example,  $dy/dx = 5x$

A differential equation contains derivatives which are either partial derivatives or ordinary derivatives. The derivative represents a rate of change, and the differential equation describes a relationship between the quantity that is continuously varying with respect to the change in another quantity. There are a lot of differential equations formulas to find the solution of the derivatives.[1,2,3]

### Order of Differential Equation

The order of the differential equation is the order of the highest order derivative present in the equation. Here some examples for different orders of the differential equation are given.

- $dy/dx = 3x + 2$  , The order of the equation is 1
- $(d^2y/dx^2)+ 2 (dy/dx)+y = 0$ . The order is 2
- $(dy/dt)+y = kt$ . The order is 1

### First Order Differential Equation

You can see in the first example, it is a first-order differential equation which has degree equal to 1. All the linear equations in the form of derivatives are in the first order. It has only the first derivative such as  $dy/dx$ , where x and y are the two variables and is represented as:



$$dy/dx = f(x, y) = y'$$

Second-Order Differential Equation

The equation which includes the second-order derivative is the second-order differential equation. It is represented as;

$$d^2y/dx^2 = f''(x) = y''$$

## II. DISCUSSION

Degree of Differential Equation

The degree of the differential equation is the power of the highest order derivative, where the original equation is represented in the form of a polynomial equation in derivatives such as  $y'$ ,  $y''$ ,  $y'''$ , and so on.[4,5,6]

Suppose  $(d^2y/dx^2) + 2(dy/dx) + y = 0$  is a differential equation, so the degree of this equation here is 1. See some more examples here:

- $dy/dx + 1 = 0$ , degree is 1
- $(y''')^3 + 3y'' + 6y' - 12 = 0$ , degree is 3
- $(dy/dx) + \cos(dy/dx) = 0$ ; it is not a polynomial equation in  $y'$  and the degree of such a differential equation can not be defined.

Note:

Order and degree (if defined) of a differential equation are always positive integers.

Types of Differential Equations

Differential equations can be divided into several types namely

- Ordinary Differential Equations
- Partial Differential Equations
- Linear Differential Equations
- Nonlinear differential equations
- Homogeneous Differential Equations
- Nonhomogeneous Differential Equations

Ordinary Differential Equation

An ordinary differential equation involves function and its derivatives. It contains only one independent variable and one or more of its derivatives with respect to the variable.

The order of ordinary differential equations is defined as the order of the highest derivative that occurs in the equation.[7,8,9] The general form of n-th order ODE is given as

$$F(x, y, y', \dots, y^n) = 0$$



## Differential Equations Solutions

A function that satisfies the given differential equation is called its solution. The solution that contains as many arbitrary constants as the order of the differential equation is called a general solution. The solution free from arbitrary constants is called a particular solution. There exist two methods to find the solution of the differential equation.

1. Separation of variables
2. Integrating factor

### III. RESULTS

Separation of the variable is done when the differential equation can be written in the form of  $dy/dx = f(y)g(x)$  where  $f$  is the function of  $y$  only and  $g$  is the function of  $x$  only. Taking an initial condition, rewrite this problem as  $1/f(y)dy = g(x)dx$  and then integrate on both sides.

Also, check: Solve Separable Differential Equations

Integrating factor technique is used when the differential equation is of the form  $dy/dx + p(x)y = q(x)$  where  $p$  and  $q$  are both the functions of  $x$  only.

First-order differential equation is of the form  $y' + P(x)y = Q(x)$ , where  $P$  and  $Q$  are both functions of  $x$  and the first derivative of  $y$ . The higher-order differential equation is an equation that contains derivatives of an unknown function which can be either a partial or ordinary derivative. It can be represented in any order. [10,11,12]

We also provide a differential equation solver to find the solutions for related problems.

#### Applications

Differential equations have several applications in different fields such as applied mathematics, science, and engineering. Apart from the technical applications, they are also used in solving many real life problems. Let us see some differential equation applications in real-time.

- 1) Differential equations describe various exponential growths and decays.
- 2) They are also used to describe the change in return on investment over time.
- 3) They are used in the field of medical science for modelling cancer growth or the spread of disease in the body.
- 4) Movement of electricity can also be described with the help of it.
- 5) They help economists in finding optimum investment strategies.
- 6) The motion of waves or a pendulum can also be described using these equations.

The various other applications in engineering are: heat conduction analysis, in physics it can be used to understand the motion of waves. The ordinary differential equation can be utilized as an application in the engineering field for finding the relationship between various parts of the bridge.

Now, go through the differential equations examples in real-life applications .





Linear Differential Equations Real World Example[13,14,15]

To understand Differential equations, let us consider this simple example. Have you ever thought about why a hot cup of coffee cools down when kept under normal conditions? According to Newton, cooling of a hot body is proportional to the temperature difference between its temperature T and the temperature T<sub>0</sub> of its surrounding. This statement in terms of mathematics can be written as:

dT/dt ∝ (T – T<sub>0</sub>).....(1)

This is the form of a linear differential equation.

Introducing a proportionality constant k, the above equation can be written as:

dT/dt = k(T – T<sub>0</sub>) .....(2)

Here, T is the temperature of the body and t is the time,

T<sub>0</sub> is the temperature of the surrounding,

dT/dt is the rate of cooling of the body

A differential equation is an equation with one or more variables (unknowns) and some of their derivatives. That means the differential equation defines the relationship between variables and their derivatives.

In this article, you will learn the definition of the order and degree of differential equations and how to find the order and degree of a given differential equation, along with solved examples.

Order of Differential Equation

Differential Equations are classified on the basis of the order. The order of a differential equation is the order of the highest derivative (also known as differential coefficient) present in the equation.[16,17,18]

IV. CONCLUSION

It is not possible every time that we can find the degree of a given differential equation. The degree of any differential equation can be found when it is in the form of a polynomial; otherwise, the degree cannot be defined.

Suppose in a differential equation dy/dx = tan (x + y), the degree is 1, whereas for a differential equation tan (dy/dx) = x + y, the degree is not defined. These types of differential equations can be observed with other trigonometry functions such as sine, cosine and so on.[19,20,21]

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