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Hydro Magnetic Mixed Convection And Mass Transfer Flow of a Viscoelastic Fluid through a Porous Medium Confined in a Vertical Channel: a Comparison between Crank-Nicolson Method and Laplace Transform Technique

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ABSTRACT: The present paper deals with mass transfer effect on unsteady mixed convection flow of an incompressible, electrically conducting viscoelastic second order Rivlin-Ericksen fluid through a porous medium confined in a vertical channel under the influence of transversely applied uniform magnetic flied. The channel walls are maintained at constant but different temperatures. The Boussinsq approximation is invoked, so that the density variation is retained in the buoyancy term only. Solution for the velocity distribution, the temperature field and the concentration field as well as the expressions for the skin-friction, the Nusselt number and the Sherwood number are derived by the use of Laplace transform technique and the Crank-Nicolson method. These are evaluated numerically for different values of the parameters and presented graphically. Excellent agreement is observed between the graphs obtained by both the methods.

KEYWORDS: Rivlin-Ericksen fluid, Viscoelastic Fluid, Vertical Channel, Laplace transform

I. INTRODUCTION

In nature, the flows exist, which are caused not only by the temperature differences but also by concentration of dissimilar chemical species. Natural convection induced by the simultaneous action of buoyancy forces resulting from thermal and mass diffusion is of considerable interest due to its natural occurrence and industrial applications such as geophysics, oceanography, engineering sciences, drying processes and solidification of binary alloy. Besides, the phenomenon of heat and mass transfer is also encountered in chemical process industries such as food processing, polymer production etc.

Heat transfer effects in non-Newtonian fluid flows are encountered in a wide range of engineering applications. Hot rolling, extrusion of plastics, flow in journal bearing, lubrication, flow in a shock absorber are some typical examples to name just a few (Bohme 1981, Huilgol & Phan-Thein 1997). Abundance use of these applications in the past decades has urged scientists and engineers to provide mathematical models for non-Newtonian fluid flows with heat transfer. The nonlinearity between stress and deformation rate for this kind of fluids makes it, in general, too much complex to obtain a simple mathematical model in comparison with the case for Newtonian fluid flows. This difficulty has lead researchers to investigate relatively simple non-Newtonian fluid flow models. The aim of the present paper is to extend the work of Sreekanth et al. (1999) to unsteady hydromagnetic mixed convection flow of a viscoelastic second order Rivlin-Ericksen fluid through a porous medium in a vertical channel under the influence of transversely applied uniform magnetic field. The channel walls are maintained at different but constant temperatures. Boussinesq approximation is invoked, so that the density variation is retained only in the buoyancy term. The solutions for the velocity, the temperature and the concentration field are obtained using the Laplace transform technique and the Crank-Nicolson method. Expressions for shear-stress, Nusselt number and Sherwood number are also derived. The results of the study are evaluated numerically for different values of the parameters and shown graphically. An excellent agreement is noted in the results obtained by these methods. The results obtained by the Sreekanth et al. (1999) are deduced as particular case of the present study.

II. FORMULATION OF THE PROBLEM

Based on the postulates of gradually fading memory, Rivlin-Ericksen (1955) derived the constitutive equation for an incompressible homogeneous fluid of second-order in the following form:

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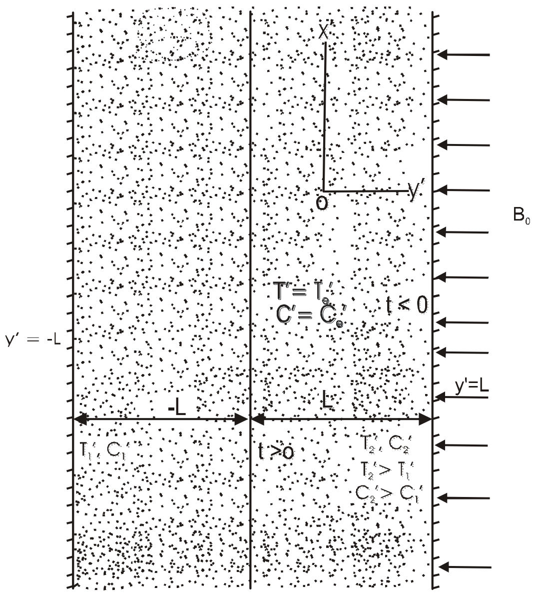

Fig. 1: Physical model and coordinate system.

$$\tau = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 , \quad (1)$$

Where τ is the Cauchy stress tensor, $-pI$ is the spherical stress due to constraint of incompressibility, μ is the dynamic viscosity, α_1, α_2 are the normal stress moduli; A_1 and A_2 are the first two Rivlin-Ericksen tensors and are defined as:

$$A_1 = (\text{grad}\, q) + (\text{grad}\, q)^T . \quad (2)$$

$$A_2 = \frac{dA_1}{dt} + A_1 (\text{grad}\, q) + (\text{grad}\, q)^T A_1 . \quad (3)$$

Here q denotes the velocity of fluid and $\frac{d}{dt}$ is the material time derivative.

The model Eq. (1) was derived by considering upto second-order approximation of retardation parameter. Dunn & Fosdick (1974) have given the range of values of material moduli μ, α_1 and α_2 as follows:

$$\mu \geq 0, \qquad \alpha_1 \geq 0, \qquad \alpha_1 + \alpha_2 = 0. \quad (4)$$

We consider the unsteady MHD mixed convective heat and mass transfer flow of an incompressible, slightly conducting viscoelastic second order Rivlin-Ericksen fluid through a porous medium confined in a vertical channel. Initially, when $t \leq 0$, i.e., at the equilibrium state, the fluid temperature and the concentration are T'_e and C'_e respectively. At $t > 0$, the temperature of one of the channel walls is instantaneously raised to T'_1 and the species concentration is raised to C'_1 such that $T'_1 > T'_e$ and $C'_1 > C'_e$. The temperature and the concentration at the second channel wall are raised (or lowered) to T'_2 and C'_2 respectively, which are thereafter maintained constant. The usual Boussinesq approximation with species concentration is invoked so that the density variations are retained in the buoyancy terms only. The viscous dissipation term in the energy equation is neglected in comparison to the heat flow by conduction and convection. A linear variation in density, temperature and concentration is assumed with ρ_e, T_e and C_e being the density, temperature and concentration of the fluid in the equilibrium state. We choose two-dimensional cartesian coordinate system (x', y') with x'-axis in the vertical direction and the walls at the distance

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$y' = \pm L$. The physical model and the coordinate system is shown in Fig.-1. As the walls of the channel are of infinite extent, the velocity, temperature and concentration are functions of y' and t' only. A uniform magnetic field of strength $B_0 \left(= \mu_e H_0\right)$ is introduced in the y'-direction, so that the velocity and the magnetic field are given by $\overline{q} = (u', 0, 0)$ and $\overline{H} = (0, H_0, 0)$ respectively. The fluid is slightly conducting and the magnetic Reynolds number is much less than unity, so that the induced magnetic field is neglected in comparison with the applied magnetic field. Besides, the foreign mass is present at low level and uniformly distributed in the flow region.

The walls temperature ratio parameter (W_T) and walls concentration ratio parameter (W_C) are very important due to their fundamental effects on the heat and mass transfer processes due to the flow of the viscoelastic fluid between the vertical walls. The parameter W_T essentially fixes the orientation of the temperature in the equilibrium state T_e' with respect to the temperature of vertical walls T_1' and T_2'. Similarity the parameter W_C is responsible for the orientation of the concentration in the equilibrium state C_e' with respect to the concentration C_1 and C_2' at the vertical walls. The case $W_T > 1$ results in $T_2' > T_1'$, whereas the case $W_T < 1$ results in $T_2' < T_e'$. Similarly, the case $W_C > 1$ implies $C_2' > C_1'$, whereas the case $W_C < 1$ implies $C_2' < C_e'$. When $W_T = 1$, the walls temperature T_1' and T_2' are equal. Also,when $W_C = 1$, the concentration at the channel walls C_1' and C_2' are equal. The case $W_T = 0$ implies that the mean of walls temperature T_1' and T_2' is equals to the initial state equilibrium temperature and the case $W_C = 0$ declares that the mean of the concentration C_1' and C_2' at the channel walls is equal to the initial state equilibrium concentration.

III. SOLUTION OF THE PROBLEM

A. *Solution by the use Laplace transform technique:*

$$\frac{\partial^2 \overline{T}}{\partial y^2} = pPr\overline{T} \cdot \qquad \frac{\partial^2 \overline{C}}{\partial y^2} = pSc\overline{C} \cdot \tag{5}$$

$$(1+\lambda p)\frac{\partial^2 \overline{u}}{\partial y^2} - (M_1 + p)\overline{u} = -Gr\overline{T} - Gm\overline{C} \cdot \tag{6}$$

The boundary conditions (13) for $t > 0$ transform to:

$$\overline{u} = 0, \quad \overline{T} = \frac{1}{p}, \quad \overline{C} = \frac{1}{p} \quad \text{at} \quad y = -1.$$

$$\overline{u} = 0, \quad \overline{T} = \frac{W_T}{p}, \quad \overline{C} = \frac{W_C}{p} \quad \text{at} \quad y = 1. \tag{7}$$

Solutions of theEquations. (4)-(6) satisfying the boundary conditions (7) are given by:

$$\overline{T} = \frac{(W_T + 1)}{2p} \frac{\cosh\sqrt{pPr}\,y}{\cosh\sqrt{pPr}} + \frac{(W_T - 1)}{2p} \frac{\sinh\sqrt{pPr}\,y}{\sinh\sqrt{pPr}} \cdot \tag{8}$$

$$\overline{C} = \frac{(W_C + 1)}{2p} \frac{\cosh\sqrt{pSc}\,y}{\cosh\sqrt{pSc}} + \frac{(W_C - 1)}{2p} \frac{\sinh\sqrt{pSc}\,y}{\sinh\sqrt{pSc}} \cdot \tag{9}$$

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$$\bar{u}=\left[\frac{A_1+B_1}{p}+\frac{A_2}{p-K_1}+\frac{A_3}{p-K_2}+\frac{B_2}{p-K_3}+\frac{B_3}{p-K_4}\right]\frac{\cosh\sqrt{\frac{M_1+p}{1+\lambda p}}y}{\cosh\sqrt{\frac{M_1+p}{1+\lambda p}}}+\left[\frac{A_4+B_4}{p}+\frac{A_5}{p-K_1}+\frac{A_6}{p-K_2}+\frac{B_5}{p-K_3}+\frac{B_6}{p-K_4}\right]\frac{\sinh\sqrt{\frac{M_1+p}{1+\lambda p}}y}{\sinh\sqrt{\frac{M_1+p}{1+\lambda p}}}$$

$$-\left[\frac{A_1}{p}+\frac{A_2}{p-K_1}+\frac{A_3}{p-K_2}\right]\frac{\cosh\sqrt{pPr}\,y}{\cosh\sqrt{pPr}}$$

$$-\left[\frac{A_4}{p}+\frac{A_5}{p-K_1}+\frac{A_6}{p-K_2}\right]\frac{\sinh\sqrt{pPr}\,y}{\sinh\sqrt{pPr}}$$

$$-\left[\frac{B_1}{p}+\frac{B_2}{p-K_3}+\frac{B_3}{p-K_4}\right]\frac{\cosh\sqrt{pSc}\,y}{\cosh\sqrt{pSc}}$$

$$-\left[\frac{B_4}{p}+\frac{B_5}{p-K_3}+\frac{B_6}{p-K_4}\right]\frac{\sinh\sqrt{pSc}\,y}{\sinh\sqrt{pSc}}. \quad (10)$$

The constant are defined in the appendix.

Using inverse Laplace transform technique, the solutions of Equations. (8)-(10) satisfying the initial and boundary conditions (7) in terms of the notations use by Jha (1998) and Singh (2005) and also calculus of residue [see Sneddon (1979)], we obtain:

$$T=\frac{(W_T+1)}{2}\sum_{r=0}^{\infty}(-1)^r\left[H_1(X_r,Pr,0,t)+H_2(Y_r,Pr,0,t)\right]+\frac{(W_T-1)}{2}\sum_{r=0}^{\infty}\left[H_1(X_r,Pr,0,t)-H_2(Y_r,Pr,0,t)\right]. \quad (11)$$

$$C=\frac{(W_C+1)}{2}\sum_{r=0}^{\infty}(-1)^r\left[H_3(X_r,Sc,0,t)+H_4(Y_r,Sc,0,t)\right]+\frac{(W_C-1)}{2}\sum_{r=0}^{\infty}\left[H_3(X_r,Sc,0,t)-H_4(Y_r,Sc,0,t)\right]. \quad (12)$$

$$u=(A_1+B_1)\frac{\cosh\sqrt{M_1}\,y}{\cosh\sqrt{M_1}}+A_2\frac{\cosh\sqrt{\frac{M_1+K_1}{1+\lambda K_1}}y}{\cosh\sqrt{\frac{M_1+K_1}{1+\lambda K_1}}}e^{K_1t}+A_3\frac{\cosh\sqrt{\frac{M_1+K_2}{1+\lambda K_2}}y}{\cosh\sqrt{\frac{M_1+K_2}{1+\lambda K_2}}}e^{K_2t}+B_2\frac{\cosh\sqrt{\frac{M_1+K_3}{1+\lambda K_3}}y}{\cosh\sqrt{\frac{M_1+K_3}{1+\lambda K_3}}}e^{K_3t}+B_3\frac{\cosh\sqrt{\frac{M_1+K_4}{1+\lambda K_4}}y}{\cosh\sqrt{\frac{M_1+K_4}{1+\lambda K_4}}}e^{K_4t}$$

$$-(-1)^n(2n+1)\pi\cos\left\{(2n+1)\frac{\pi y}{2}\right\}\frac{(1-\lambda p_{n_1})^2}{(1-\lambda M_1)}\left(\frac{A_1+B_1}{p_{n_1}}+\frac{A_2}{p_{n_1}+K_1}+\frac{A_3}{p_{n_1}+K_2}+\frac{B_2}{p_{n_1}+K_3}+\frac{B_3}{p_{n_1}+K_4}\right)e^{-p_{n_1}t}+(A_4+B_4)\frac{\sinh\sqrt{M_1}\,y}{\sinh\sqrt{M_1}}$$

$$+A_5\frac{\sinh\sqrt{\frac{M_1+K_1}{1+\lambda K_1}}y}{\sinh\sqrt{\frac{M_1+K_1}{1+\lambda K_1}}}e^{K_1t}+A_6\frac{\sinh\sqrt{\frac{M_1+K_2}{1+\lambda K_2}}y}{\sinh\sqrt{\frac{M_1+K_2}{1+\lambda K_2}}}e^{K_2t}$$

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$$+B_5 \frac{\sinh\sqrt{\frac{M_1+K_3}{1+\lambda K_3}}y}{\sinh\sqrt{\frac{M_1+K_3}{1+\lambda K_3}}} e^{K_3 t} + B_6 \frac{\sinh\sqrt{\frac{M_1+K_4}{1+\lambda K_4}}y}{\sinh\sqrt{\frac{M_1+K_4}{1+\lambda K_4}}} e^{K_4 t} + (-1)^n 2n\pi \sin(n\pi y) \frac{(1-\lambda p_{n_2})^2}{(1-M_1\lambda)} \left(\frac{A_4+B_4}{p_{n_2}} + \frac{A_5}{p_{n_2}+K_1} \right.$$

$$\left. + \frac{A_6}{p_{n_2}+K_2} + \frac{B_5}{p_{n_2}+K_3} + \frac{B_6}{p_{n_2}+K_4} \right) e^{-p_{n_2} t}$$

$$-A_1 \sum_{r=0}^{\infty} (-1)^r \left[H_5(X_r, Pr, 0, t) + H_6(Y_r, Pr, 0, t) \right]$$

$$-A_2 e^{K_1 t} \sum_{r=0}^{\infty} (-1)^r \left[H_7(X_r, Pr, K, t) + H_8(Y_r, Pr, K, t) \right] - A_3 e^{K_2 t} \sum_{r=0}^{\infty} (-1)^r \left[H_9(X_r, Pr, K_2, t) + H_{10}(Y_r, Pr, K_2, t) \right]$$

$$-A_4 \sum_{r=0}^{\infty} \left[H_5(X_r, Pr, 0, t) - H_6(Y_r, Pr, 0, t) \right]$$

$$-A_5 e^{K_1 t} \sum_{r=0}^{\infty} \left[H_7(X_r, Pr, K_1, t) - H_8(Y_r, Pr, K_1, t) \right]$$

$$-A_6 e^{K_2 t} \sum_{r=0}^{\infty} \left[H_9(X_r, Pr, K_2, t) - H_{10}(Y_r, Pr, K_2, t) \right] - B_1 \sum_{r=0}^{\infty} (-1)^r \left[H_{11}(X_r, Sc, O, t) + H_{12}(Y_r, Sc, O, t) \right]$$

$$-B_2 e^{K_3 t} \sum_{r=0}^{\infty} (-1)^r \left[H_{13}(X_r, Sc, K_3, t) + H_{14}(Y_r, Sc, K_3, t) \right] - B_3 e^{K_4 t} \sum_{r=0}^{\infty} (-1)^r \left[H_{15}(X_r, Sc, K_4, t) + H_{16}(Y_r, Sc, K_4, t) \right]$$

$$-B_4 \sum_{r=0}^{\infty} \left[H_{11}(X_r, Sc, 0, t) - H_{12}(Y_r, Sc, 0, t) \right] - B_5 e^{K_3 t} \sum_{r=0}^{\infty} \left[H_{13}(X_r, Sc, K_3, t) - H_{14}(Y_r, Sc, K_3, t) \right]$$

$$-B_6 e^{K_4 t} \sum_{r=0}^{\infty} \left[H_{15}(X_r, Sc, K_4, t) - H_{16}(Y_r, Sc, K_4, t) \right]. \quad (13)$$

where $F(Z_1, Z_2, Z_3, Z_4) = \frac{1}{2}\left[\exp\left(-Z_1\sqrt{Z_2 Z_3}\right) erfc\left(\frac{Z_1\sqrt{Z_2}}{2\sqrt{Z_4}} - \sqrt{Z_3 Z_4} \right) + \exp\left(Z_1\sqrt{Z_2 Z_3}\right) erfc\left(\frac{Z_1\sqrt{Z_2}}{2\sqrt{Z_4}} + \sqrt{Z_3 Z_4} \right) \right],$

$X_r = 2r + 1 - y$ and $Y_r = 2r + 1 + y$.

B. Solution by the use of Crank-Nicolson scheme:

The governing equations (6)-(8) are unsteady, coupled and non-linear with initial and boundary conditions (9). They are solved by an implicit finite difference scheme of Crank-Nicolson method as described by Cheng & Minkowycz (1977). By the use of Crank-Nicolson formula, we obtain:

$$-r\lambda U_{i-1,\, j+1} + (2r\lambda + k) U_{i,\, j+1} - r\lambda U_{i+1,\, j+1} = D_{i,\, j}. \quad (14)$$

$$-rT_{i-1,\, j+1} + (2r + Pr) T_{i,\, j+1} - rT_{i+1,\, j+1} = J_{i,\, j}, \quad (15)$$

$$-rC_{i-1,\, j+1} + (2r + Sc) C_{i,\, j+1} - rC_{i+1,\, j+1} = N_{i,\, j}. \quad (16)$$

where $D_{i,\,j} = r(k-\lambda) U_{i-1,\,j} + \left[\lambda r - 2kr + k - h^2 kr M_1 \right] U_{i,\,j}$

$+r(k-\lambda) U_{i+1,\,j} + h^2 kr \left[C_{i,\,j} + G T_{i,\,j} \right].$

$J_{i,\,j} = -rT_{i-1,\,j} + (r - Pr) T_{i,\,j} + rT_{i+1,\,j}.$

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$$N_{i,j} = -rC_{i-1,j} + 2(r - Sc)C_{i,j} + rC_{i+1,j}. \quad r = \frac{k}{h^2}.$$

In Equations. (14)-(15), the quantities *h* and *k* are mesh sizes along *y*-direction and time direction respectively. In Eq. (14) and (15) we take $i = 2,3,4,......,10$ and use the following initial and boundary conditions:

$t \le 0,\ \ u = 0,\ \ T = 0,\ \ C = 0$ for all y.

$t > 0,\ \ u = 0,\ \ T = 1,\ \ C = 1$ at $y = -1$.

$$u = 0,\ \ T = W_T,\ \ C = W_C \ \text{ at } y = 1. \tag{17}$$

By the use of above stated symbols, initial and boundary conditions, a tridiagonal system of equations is obtained. The system of equations is solved by the use of the software Mathematica. The region of integration is considered by taking the mesh sizes along *y*-direction and *t*-direction respectively. The value $y_{max}\,(=14.0)$ corresponds to $y = \infty$, which lie outside the momentum, thermal and concentration boundary layers. An appropriate mesh size $\Delta y = 0.25$ and time step $\Delta t = 0.01$ is considered for calculations.

IV. RESULTS AND DISCUSSION

The effect of mass transfer on the hydromagnetic mixed convection flow of a viscoelastic second order Rivlin-Ericksen fluid through a porous medium confined in a vertical channel is studied. The non-dimensional equations for momentum, energy and concentration are expressed in (5)-(7). The solutions for temperature, concentration and velocity distributions are shown in (15)-(17). The heat transfer rate, mass transfer rate and shear-stress at the channel walls are expressed in (18)-(20). The solutions are also obtained by two methods; namely, by analytical method using Laplace transform technique and by the use of Crank-Nicolson method. The results obtained from both the methods are in excellent agreement. The solutions show that the velocity in the channel is governed by the walls temperature parameter (W_T), the walls concentration parameter (W_C), the magnetic parameter (*M*), the permeability parameter (*k*), the viscoelastic parameter (λ), the Grashof number (*Gr*) and the Solutal Grashof number (*Gm*). The temperature and concentration distributions are governed by the Prandtl number (*Pr*) and the Schmidt number (*Sc*) respectively, In order to observe the mystery and physical insight of the problem, effects of the walls temperature ratio, walls concentration ratio, viscoelastic parameter, magnetic field, permeability parameter and buoyancy parameter on the velocity field are observed. The effects of Prandtl number (*Pr*) and walls temperature ratio on the temperature distribution and that of Schmidt number (*Sc*) on the concentration field are also observed. Numerical calculations have been made and presented in the form of figures. An important aspect of the problem is that the values of the Grashof number (Gr) are chosen for cooling ($Gr > 0$) case of the channel walls, which usually occurs in engineering and industrial applications [Rosa (1968), Blums (1987)]. Numerical values of the parameters are chosen following authors of the field, namely, Sreekanth et al. (1999) and Singh (2008). To be realistic, the values of the Prandtl number (*Pr*) are chosen to be 10, 15, 20 and 25, whereas the values of Schmidt number (*Sc*) are chosen as 50, 75, 100 and 125, which correspond to viscoelastic fluids of second order. Throughout the discussion it is assumed that $T_2' > T_1'$ and $T_1' > T_e'$ if not specified otherwise. It is observed that the results obtained by both the methods are in excellent agreement.

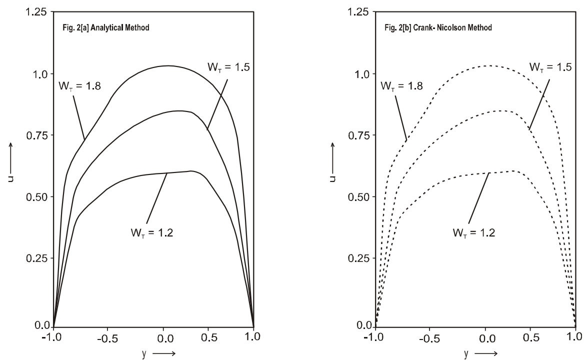

Fig. 2 [a],[b]: Effect of walls temperature ratio $W_T > 1$ on the velocity at M = 0.5, k = 2.0, λ = 1.5, G_r = 4.0, G_m = 4.0, t = 1.0, W_c = 1.2, P_r = 10 and S_c = 50.

Fig.-2a and Fig.-2b are intended to illustrate the velocity profiles of the viscoelastic fluid for different values of the walls temperature ratio parameter (W_T) against *y*, which are drawn on the basis of the numerical values introduced in the analytical solution by the use of Laplace transform technique and the solution obtained by Crank-Nicolson scheme respectively. The profiles are drawn for the given values of $W_C = 1.2$, $M = 0.5$, $k = 0.2$, $\lambda = 0.5$, $Gr = 4.0$, $Gm = 4.0$, $Pr = 10$, $t = 1.0$ and $Sc = 50$. It is observed that when $W_T > 1$, the velocity of the viscoelastic second order Rivlin-Ericksen fluid is maximum at the wall with temperature T_2' ($y = 1$), which gradually decreases as we move towards the wall with temperature T_1' ($y = -1$). Besides, the velocity profiles of both the methods are in excellent agreement. Physically, $W_T > 1$ implies $T_2' > T_1' > T_e'$. Therefore, the ambient fluid is being heated by both the walls i.e., by the wall at $y = -1$ as well as by the wall at $y = 1$. Thus, the velocity in the vicinity of the wall at $y = 1$ is more enhanced in comparison with the velocity in the neighborhood of the channel wall at $y = -1$.

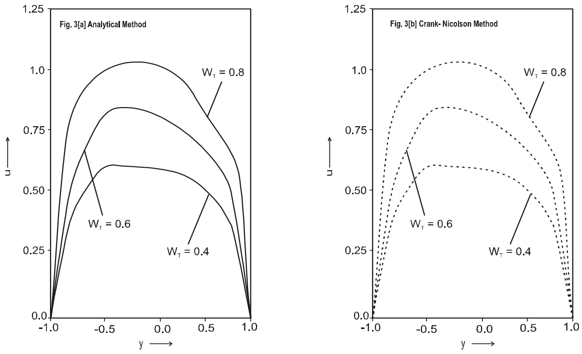

Fig. 3 [a], [b]: Effect of walls temperature ratio W_T ($0 < W_T < 1$) on the velocity at M = 0.5, k = 2.0, λ = 1.5, G_r = 4.0, G_m = 4.0, t = 1.0, W_c = 1.2, P_r = 10 and S_c = 50.

Fig.-3a and Fig.-3b show the velocity profiles of the second order viscoelastic fluid against *y* for different values of the walls temperature ratio parameter $0 < W_T < 1$ based on the Laplace transform technique and the Crank-Nicolson method respectively at fixed values $M = 0.5$, $k = 0.2$, $\lambda = 0.5$, $Gr = 4.0$, $Gm = 4.0$, $t = 1.0$, $W_C = 1.2$, $Pr = 10$ and $Sc = 50$. It is noticed that when $0 < W_T < 1$, the velocity is maximum in the vicinity of the wall ($y = -1$) which is at temperature T_1'. The velocity gradually decreases as we move towards the wall with temperature T_2' ($y = 1$). The physics behind this phenomenon is hidden in the fact that $W_T < 1$ implies $T_1' > T_2' > T_e'$. Therefore, the ambient fluid is being heated more effectively by the wall ($y = -1$) with temperature T_1' in comparison with the wall at temperature $T_2'(y = 1)$. As such, the enhanced velocity is observed at the wall $y = -1$. Besides, it is observed that the profiles obtained by both the methods are in excellent agreement.

V. CONCLUSION

This paper focuses on the laminar heat and mass transfer flow of an electrically conducting non-Newtonian second order viscoelastic Rivlin-Ericksen fluid in a vertical channel under the influence of transversely applied uniform magnetic field. The flow occurs between parallel walls at different walls temperature and species concentration. Analytical solutions of the governing boundary layer partial differential equations, which are highly non-linear, have been obtained by two methods; namely, by the use of Laplace transform technique and the results of the study are as follows:

The results obtained by the Laplace transform technique and by Crank-Nicolson scheme are in excellent agreement.The velocity in the channel increases with increase in the walls temperature ratio (W_T) when $W_T > 1$ and $W_T < 1$. The reverse flow exists, when the walls temperature ratio $W_T = 1$ and $W_T = -1$, whereas walls temperature ratio $W_T = 0$ prevails velocity symmetrical to the channel walls.An increase in the walls concentration ratio (.$W_C > 1$)

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decreases the velocity. An increase in the viscoelastic parameter (λ) or magnetic parameter (*M*) results in a decrease in the velocity. The velocity increases with increase in the buoyancy parameter (*Gr*) or permeability parameter (*k*). An increase in walls temperature ratio (W_T) or Prandtl number (*Pr*) decreases the temperature. An increase in walls concentration ratio (W_C) and Schmidt number (*Sc*) decrease the concentration.

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