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Quadratic Diophantine Equations of the Form $2xy = n(x+y)$ and $3xy = n(x+y)$

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ABSTRACT: In this paper ,the quadratic diophantine equations of the form $2xy = n(x+y)$ and $3xy = n(x+y)$ have been considered. An attempt has been made to obtain non-zero distinct integer solutions to the above quadratic diophantine equations through elementary methods.

KEYWORDS: Quadratic equation , Non-homogeneous quadratic , Integer solutions

I. INTRODUCTION

It is well-known that diophantine equations , homogeneous or non- homogeneous , have aroused the interest of many mathematicians. There is a vast general theory for quadratic equations . In particular, the theory of quadratic equations in two variables is a very developed theory but still an important topic of current research. For example, [1-7] exhibits sets of integer solutions to the second degree Diophantine equations of the form $Ax^2 - By^2 = C$,where A, B, C take special values. This paper aims at finding integer solutions to second degree Diophantine equations of the form $2xy = n(x+y)$ and $3xy = n(x+y)$,where n is any non-zero positive integer. Different sets of integer solutions to the above equations are respectively obtained through employing elementary methods.

Method of analysis

Diophantine Equation of the form $2xy = n(x+y)$

The non-homogeneous second degree Diophantine to be solved is

$$2xy = n(x+y) \tag{1}$$

The process of obtaining different sets of integer solutions to (1) is illustrated below :

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Illustration 1:

Introduction of the transformations

$$x = k\,y\,,k \ge 1 \tag{2}$$

in (1) leads to

$$2\,k\,y = n\,(k+1)$$

which is satisfied by

$$n = 2\,k\,s\,,y = (k+1)\,s \tag{3}$$

From (2) , we have

$$x = k\,(k+1)\,s \tag{4}$$

Thus , (3) & (4) represent the integer solutions to (1) .

Illustration 2 :

Introducing the transformations

$$x = u + v\,,y = u - v\,,u \ne v \ne 0 \tag{5}$$

in (1) , we have

$$u^2 - n\,u - v^2 = 0$$

Treating the above equation as quadratic in u and solving for u , we get

$$u = \frac{n \pm \sqrt{n^2 + 4\,v^2}}{2} \tag{6}$$

It is possible to choose the values for n & v so that the square-root on the

R.H.S. of (6) is eliminated and the corresponding values for u are obtained.

In view of (5) , the respective values of x & y satisfying (1) are found.

For simplicity and brevity ,the integer solutions to (1) thus obtained are presented

in Table 1 as follows :

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Table 1 – Integer solutions

n	x	y
$3s$	$6s$	$2s$
$3s$	s	$-3s$
$2s^2-2, s\geq 1$	$2s^2+2s$	$2s^2-2s$
$2s^2-2, s\geq 1$	$2s-2$	$-2s-2$
$p^2-q^2, p\geq q\geq 0$	$p(p+q)$	$p(p-q)$
$p^2-q^2, p\geq q\geq 0$	$q(p-q)$	$-q(p+q)$
$8pq$	$4p(p+q)$	$4q(p+q)$
$8pq$	$4q(p-q)$	$4p(q-p)$

Diophantine Equation of the form $3xy = n(x+y)$

The non-homogeneous second degree Diophantine to be solved is

$$3xy = n(x+y) \tag{7}$$

The process of obtaining different sets of integer solutions to (7) is illustrated below :

Illustration 3:

Introduction of the transformations

$$x = ky, k\geq 1 \tag{8}$$

in (7) leads to

$$3ky = n(k+1)$$

which is satisfied by

$$n = 3ks, y = (k+1)s \tag{9}$$

From (8) , we have

$$x = k(k+1)s \tag{10}$$

Thus , (9) & (10) represent the integer solutions to (7) .

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Illustration 4 :

Introducing the transformations (5) in (7) ,we have

$$3u^2 - 2nu - 3v^2 = 0$$

Treating the above equation as quadratic in u and solving for u , we get

$$u = \frac{n \pm \sqrt{n^2 + 9v^2}}{3} \tag{11}$$

It is possible to choose the values for n & v so that the square-root on the

R.H.S. of (11) is eliminated and the corresponding values for u are obtained.

In view of (5) , the respective values of x & y satisfying (7) are found.

For simplicity and brevity ,the integer solutions to (7) thus obtained are presented

in Table 2 as follows :

Table 2 – Integer solutions

n	x	y
$9r^2 - s^2, 3r \geq s \geq 0$	$6r^2 + 2rs$	$6r^2 - 2rs$
$18rs$	$6r^2 + 6rs$	$6s^2 + 6rs$
$18s^2 + 18s + 4$	$12s^2 + 14s + 4$	$12s^2 + 10s + 2$
$6s^2 + 6s$	$4s^2 + 6s + 2$	$4s^2 + 2s$
$6s^2 + 6s$	$2s$	$-2 - 2s$
$2s^2 + 2s - 4$	$2s - 2$	$-2s - 4$
$4s$	$4s$	$2s$
$12s$	$2s$	$-4s$

It is worth to mention that , in [8] ,the authors have presented integer solutions when n takes particular values.. Here, we have exhibited the integer solutions corresponding to other values of n also.

II. CONCLUSION

In this paper, an attempt has been made to obtain non-zero distinct integer solutions to the above quadratic Diophantine equation through elementary methods. One may search for the integer solutions to other forms of second degree equations with multiple variables.

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